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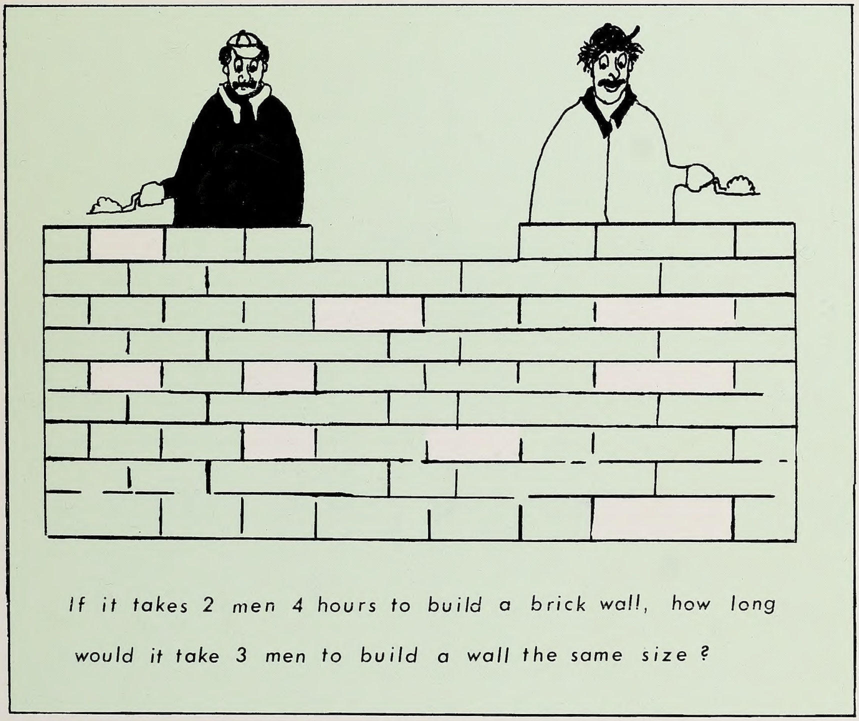
PRACTICAL MATHEMATICS



If it takes 2 men 4 hours to build a brick wall, how long would it take 3 men to build a wall the same size ?

RATIO and PROPORTION

module 4

ALBERTA CORRESPONDENCE SCHOOL
ALBERTA EDUCATION
BARRHEAD, ALBERTA

Practical Mathematics

Module 4

RATIO AND PROPORTION

Distance Learning

Alberta EDUCATION

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IMPORTANT!

PLEASE READ THIS INTRODUCTION BEFORE BEGINNING YOUR FIRST LESSON.

Content and Purpose of Course

In its entirety, this Practical Mathematics Series consists of eight modules, each containing from four to seven lessons. The titles of these modules are as follows:

Module 1: Whole Numbers
Module 2: Fractional Numbers
Module 3: Decimal Numbers
Module 4: Ratio and Proportion
Module 5: Measurement
Module 6: Introductory Algebra
Module 7: Intermediate Algebra
Module 8: Trigonometry

Taken individually, these modules should provide a person with instruction and practice in specific areas in which he finds that his mathematical background is weak.

The first five modules, taken as a unit, should equip a person to handle most practical mathematical problems encountered in day-to-day living. The next two modules cover the basics of algebra required for subsequent vocational mathematics courses. The last module covers the basics of trigonometry.

The series, Practical Mathematics, was also developed with the needs of prospective tradesmen in mind. Modules 1-5 inclusive were designed to prepare students mathematically for certain apprenticeship programs. In other words, if a person completes the first five modules of this series, he should be prepared to write the mathematics section of the examination set by the Apprenticeship Board for entrance into the following apprenticeship programs:

Agricultural Mechanic
Appliance Serviceman
Auto Body Mechanic
Baker
Bricklayer
Carpenter
Cook
Floorcovering Mechanic
Gasfitter
Glassworker
Heavy Duty Mechanic
Insulator
Lather
Machinist
Millwright
Motor Mechanic
Painter and Decorator
Partsman
Plasterer
Plumber
Refrigerator Mechanic
Roofer
Sheet Metal Mechanic
Steamfitter
Steel Fabricator
Tile Setter
Welder

If a person completes Modules 1 to 7 inclusive, he should be prepared to write the Apprenticeship Board's mathematics examination for entrance into the apprenticeship programs listed below:

Communication Electrician
Electrician
Electrical Mechanic
Instrument Mechanic
Power Electrician
Radio Technician

There is no prescribed textbook for this course. All the explanatory material and exercises are provided in the lessons.

Self-Correcting Exercises

A number of self-correcting exercises appear in the lessons in this course. You should work through each of these exercises carefully and when you have completed the **entire** exercise, check your answers with those provided at the end of the lesson. Correct any errors that you have made in these exercises. If you do not understand some of the answers, ask your correspondence teacher about them. It is important that you handle these self-correcting exercises properly as they help prepare you for the assignments that follow.

Inventory Tests

Inventory tests appear at the beginning of Modules 2-5 inclusive. These lessons are designed to test the student's grasp of basic skills which should be mastered before additional lessons are attempted. Complete each inventory test and send it in for correction before beginning any other lessons in that module.

Optional Sections

Optional sections occur in some lessons in this course. Students not progressing beyond Module 5 are not required to complete these sections, but may do so if they wish. Students who plan on taking Modules 6, 7 and 8 **must** complete these optional sections.

Dear Student:

As of June 1, 1989 you will not be required to send in any lessons in Practical Mathematics (Modules 1 - 5). At the back of the module you will find a booklet of answers and solutions.

Study the lesson notes very carefully, work on the exercises and then check your work by using this booklet of answers and solutions. If, however, you experience difficulty in doing some questions you may telephone the Alberta Correspondence School at 674-5333. Students who reside in Alberta may call the school free of charge through the local R.I.T.E. number in your area or the Zenith number 22333 (if there is no R.I.T.E. centre).

When you have completed your home study and feel confident that you know the material then you may proceed onto Mathematics 14 (if you require credits) or into Practical Mathematics (Modules 6, 7 and 8) or Basic Algebra and Geometry (if you require an academic mathematics course). However, if you are undecided which route to take then write us a letter or telephone us by using the above number.

We wish you success in this course.

Ronald Billey

Ron Billey
Vice-Principal
Mathematics Science Department

MODULE 4: RATIO AND PROPORTION

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Practical Mathematics
Module 4
Lesson 1

Learning mathematics is like building a house — the foundation must be laid before the walls can go up. If the foundation is weak, the house may collapse. The operations with whole numbers, fractional numbers, and decimal numbers form the foundation of mathematics. Let's see how well you remember your foundational skills.

Please complete the following test and send it in for correction as lesson 1 of this module.

INVENTORY TEST — Whole Numbers, Fractional Numbers, and Decimal Numbers

Part A

1. Give the missing word and number.

EXAMPLE:
27.64 The 4 in the hundredths place represents $(4 \times \frac{1}{100})$.

(a) 356.742
The 3 in the ____________ place represents ____________.
The 7 in the ____________ place represents ____________.

(b) 8743.5697
The 8 in the ____________ place represents ____________.
The 9 in the ____________ place represents ____________.

(c) 0.54826
The 2 in the ____________ place represents ____________.
The 6 in the ____________ place represents ____________.

2. Represent each of the following sums as a whole number or a decimal number.

(a) $6\ (10) + 4\ (1) + 3\left(\frac{1}{10}\right)$ = 64.3

(b) $7(1000) + 3(10)$ = ____________

(c) $5000 + 200 + 30 + 6 + \frac{3}{10} + \frac{7}{100} + \frac{5}{1000}$ = ____________

(d) $80{,}000 + 300 + 5$ = ____________

(e) $7(10{,}000) + 5(1000) + 0(100) + 3(10) + 5(1) + 4\left(\frac{1}{10}\right) + 6\left(\frac{1}{100}\right) + 2\left(\frac{1}{1000}\right)$ = ____________

(f) $1(10,000) + 1\left(\frac{1}{10,000}\right) =$ ____________

(g) $7 + \frac{3}{1000} =$ ____________

3. Write each of the following in numerals.

EXAMPLE: Ninety-nine ten-thousandths 0.0099

(a) Four hundred nine thousandths ____________

(b) One hundred thousand, seventeen ____________

(c) Four million, seven hundred sixty-five thousand, two hundred nine and six hundredths ____________

(d) Ten thousand, four hundred ____________

(e) One hundred forty-five ten-thousandths ____________

4. Tell whether or not the following pairs of fractions are equivalent. (Write "E" for the pairs which are equivalent, and "N" for those pairs which are not equivalent.)

	"E" or "N"		"E" or "N"
(a) $\frac{5}{10}, \frac{25}{50}$	E	(b) $\frac{12}{25}, \frac{48}{100}$	
(c) $\frac{1}{9}, \frac{9}{17}$		(d) $\frac{9}{10}, \frac{91}{101}$	
(e) $\frac{2}{26}, \frac{1}{13}$		(f) $\frac{20}{36}, \frac{4}{9}$	

5. Write each of the following as a mixed number. (Make sure the fraction is in the simplest form.)

(a) $6.4 = 6\frac{4}{10} = 6\frac{2}{5}$

(b) $17.61 =$

(c) $\frac{19}{7} =$

(d) $919.7154 =$

(e) $\frac{328}{26} =$

(f) 6.625 =

(g) 40.0352 =

(h) 5.21746 =

(i) 81.558 =

(j) $\frac{3003}{12}$ =

6. Place the correct symbol (<, >, or =) in each blank.

(a) 0.78 ______ 0.87

(b) 0.9 + 0.1 ______ 1.1

(c) 5 ______ $3\frac{3}{4}$

(d) 9.001 ______ 8.999

(e) 0.3 ______ 0.299

(f) $1\frac{1}{2}$ ______ 1.75

(g) 12 ______ $\frac{36}{3}$

(h) 6.750 ______ 6.75

(i) 0.018 ______ 0.02

(j) 0.34 ______ 0.43

7. Find the answers without any written computation.

(a) 5.07 × 10 = 50.7

(b) 13.29 × 10,000 = ______

(c) 5.832 ÷ 100 = ______

(d) $\frac{379.6}{10}$ = ______

(e) 17.41 × 100,000 = ______

(f) 8,320.7 × 100 = ______

(g) $\frac{29.325}{10{,}000}$ = ______

(h) 393.42 ÷ 1,000,000 = ______

(i) 0.107 ÷ 1000 = ______

(j) 0.9 × 1000 = ______

8. Write each of the following fractions as decimals. (Remember to show as many decimal places as there are zeros in the denominator.)

(a) $\frac{5}{100}$ = 0.05

(b) $\frac{35}{10,000}$ = ________

(c) $\frac{11}{10,000}$ = ________

(d) $\frac{225}{100,000}$ = ________

(e) $\frac{25}{1000}$ = ________

(f) $\frac{29}{10}$ = ________

(g) $\frac{98}{100}$ = ________

(h) $\frac{5572}{1000}$ = ________

(i) $\frac{3}{10}$ = ________

(j) $\frac{400}{10}$ = ________

9. Write each of the following decimals as a fraction or as a mixed number. Express the answer in simplest form.

(a) 0.072 = $\frac{72}{1000} = \frac{9}{125}$

(b) 0.07 = ________

(c) 0.143 = ________

(d) 0.32 = ________

(e) 2.5 = ________

(f) 1.03 = ________

(g) 0.9 = ________

(h) 6.041 = ________

(i) 12.75 = ________

(j) 1.75 = ________

10. Round each of the following numbers as specified.

(a) 99.6 rounded to the nearer whole number is 100.

(b) 1.25 rounded to the nearer tenth is ________.

(c) 1328 rounded to the nearer hundred is ________.

(d) 69.3 rounded to the nearer whole number is ________.

(e) 0.227 rounded to the nearer hundredth is ________.

(f) 589.7 rounded to the nearer whole number is ________.

(g) 0.993 rounded to the nearer tenth is ________.

(h) 1,532,495 rounded to the nearer thousand is ________.

(i) 1.374 rounded to the nearer hundredth is ________.

(j) 6.9537 rounded to the nearer thousandth is ________.

Part B

1. Arrange each set of terms in a column in the space provided below, then add. (Add zeros where necessary.) Place the sum in the blank.

(a) 1.69 + 0.582 + 60.7 + 85.5 = 148.472

(b) 0.039 + 17.5 + 9.25 + 0.675 = ____________

(c) 2.6831 + 98.567 + 840.5 = ____________

(d) 0.34 + 0.8 + 18 + 0.25 = ____________

(a)	(b)	(c)	(d)
1.690 0.582 60.700 85.500 148.472			

2. Subtract. Add zeros if necessary.

(a) 1.896
0.9

(b) 9.07
0.004

(c) 25.506
10.9

(d) 43.2 - 5.63 = ____________

(e) 198 - 56.987 = ____________

3. Place the decimal point correctly in each product. Add zeros where necessary.

(a) 224 × 2.4 = 5376

(b) 68.8 × 0.45 = 30960

(c) 2.24 × 0.024 = 5376

(d) 14.7 × 0.13 = 1911

(e) 0.07 × 0.004 = 28

(f) 0.361 × 0.057 = 20577

4. Find the products using as little written work as possible.

(a) 3.4 × 0.008 = ____________

(b) 207 × 0.04 = ____________

(c) 30 × 0.46 = ____________

(d) 1.7 × 0.8 = ____________

(e) 2.1 × 0.6 = ____________

(f) 1.5 × 0.07 = ____________

5. Multiply. Then state the product to the nearer cent.

(a) 348 × \$0.247 = \$85.96 (to nearer cent)

```
   348
 0.247
------
  2436
 1392
 696
------
85.956
```

(b) 985.8 × \$0.93 = ____________ (to nearer cent)

(c) 30,852 × \$0.068 = ____________ (to nearer cent)

6. Find the quotient of each of the following.
(Divide until you obtain a remainder of zero.)

(a) 49.13 ÷ 17 = ____________

(b) 11.52 ÷ 0.24 = ____________

(c) 0.722 ÷ 1.9 = ____________

(d) 12.167 ÷ 23 = ____________

(e) 4.394 ÷ 0.013 = ____________

(f) 522.72 ÷ 0.726 = ____________

7. Divide to 3 decimal places and then state the quotient to the nearer cent.

(a) $99.28 ÷ 4.86 = $20.43 (to nearer cent)

20.427
4.86‸) 99.28‸000
972
208
0
2080
1944
1360
972
3880
3402
478

(b) $41.58 ÷ 1.8 = ________ (to nearer cent)

(c) $10.92 ÷ 0.2553 = ________ (to nearer cent)

(d) $56.80 ÷ 0.167 = ________ (to nearer cent)

8. Change each fraction to a terminating or a repeating decimal. (Divide until a zero remainder is obtained or the remainder repeats.)

(a) $\frac{7}{32}$ = 0.21875

.21875
32) 7.00000
64
60
32
280
256
240
224
160
160
0

(b) $\frac{11}{6}$ = ________

(c) $\frac{9}{16}$ = ____________ (d) $\frac{5}{12}$ = ____________

(e) $\frac{7}{18}$ = ____________ (f) $\frac{28}{25}$ = ____________

9. Simplify each of the following:

(a) $71\overline{)\,236}$

(b) $\begin{array}{r} 905 \\ \times\ \underline{509} \end{array}$

236 ÷ 71 = ____________
(to 2 decimal places)

(c) $\begin{array}{r} 7001 \\ -\ \underline{309} \end{array}$

(d) 36.09 - 21.6 = ____________

(e) $\begin{array}{r} 16.325 \\ \times\ \underline{0.68} \end{array}$

(f) $123 \times 1.96 =$ __________

(g) $3.195 \div 1.2 =$ __________ (rounded to nearer tenth)

(h) $\frac{1}{2} + \frac{5}{6} + \frac{1}{3} =$ __________

(i) $\frac{1}{4} \div \frac{7}{16} =$ __________

(j) $3 - 2\frac{3}{8} =$ __________

(k) $1\frac{5}{8} \times 9 \times \frac{1}{2} =$ __________

(l) $1.356 \div 10{,}000 =$ __________

10. Write each of the following in symbols, then simplify. (Add zeros to decimal numbers where necessary.)

(a) Subtract 5 hundredths from 2 and 1 tenth.

$$\begin{array}{r} 2.10 \\ -\underline{0.05} \\ 2.05 \end{array}$$

(b) Add the following numbers: seven hundred eighty-five; two hundred ninety; one hundred seven; forty-four; six.

(c) Add: 9 hundredths; 152 thousandths; 8 tenths.

(d) Add seven hundred and thirty-three thousandths to three hundred fifty-nine and sixty-five hundredths.

(e) Subtract three hundred one ten-thousandths from five and six tenths.

(f) Subtract five and three eighths from ten and one third. (Since these are mixed numbers, the decimal point should be omitted.)

Part C

Solve each of the following problems. Show your calculations and make a final statement for each problem.

1. How many shelves, each 1.25 metres long, can be cut from a board 5.5 metres long? (Disregard the width of the saw cut.)

2. What is the greatest number of pieces of wire 5.5 inches long that can be cut from 225 feet of wire?

3. A plumber lays a drain pipe to a slope of 0.12" for each foot of length of pipe. What is the total drop in the pipe in a distance of 50'?

4. Automobiles are test driven for speed and gas consumption. A new model drives 292.85 km on 32.3 L of gas. How many kilometres does the car drive on each litre? (Round the answer to one decimal place.)

5. Find the cost of 52.5 square metres of cushioned vinyl floor covering. The cost of one square metre is $9.25. (Round answer to nearer cent.)

"per"

6. A steel rod weighs 1.46 kg /dm. Find the cost (to the nearer cent) of 1000 dm of rod, if the cost of the rod is $19.26 per 100 kg.

Part D (OPTIONAL)

1. Complete the following exercise, giving answers in simplest form.

(a) $$\frac{8\frac{3}{7}}{5\frac{6}{35}} =$$

(b) $$\frac{\frac{5}{6} + \frac{3}{4}}{\frac{1}{2}} =$$

(c) $$\frac{5 + \left(\frac{1}{5} \times 10\right)}{\frac{2}{3} - \frac{1}{2}} =$$

(d) $$\frac{\frac{3}{4} \times 12}{5 \div \frac{2}{5}} =$$

2. Find the sum or difference of the following measurements. (Remember to round each measurement to the precision of the measure with the least number of decimal places.)

(a) Add: 17.459 cm; 22.637 cm; 0.75 cm; 31.396 cm; 6.95 cm.

(b) Subtract: 7.35 cm from 13.016 cm.

3. State the number of significant digits in each of the following measurements.

Measurement	No. of Sig. Digits
1.007 km	__________
29.10 cm	__________
306 cm	__________
0.005 dm	__________
69.0 mm	__________

4. Find the area of a floor that measures 12.8 m by 9.75 m. Express your answer to the proper number of significant digits.

Area = length × width

= ________ × ________

= ________

= ________ m^2
(rounded to ______ significant digits)

The area of the floor is ____________________ m^2.

End of Lesson 1

Practical Mathematics
Module 4
Lesson 2

RATIO AND PERCENT

Topic One: Ratios

A RATIO compares two numbers by division. It compares like objects.

EXAMPLES:

1. For every $20 Peter spends, William spends $30.

   (a) Ratio: $\frac{20}{30}$ (This is read "twenty over thirty".)
   Explanation: This expresses the ratio of the amount Peter spends to that which William spends.

   (b) Ratio: $\frac{30}{20}$ (This is read "thirty over twenty".)
   Explanation: This is the ratio of the amount William spends to that which Peter spends.

2. At a pie-eating contest, John eats 4 pies in the same time that Henry eats 6 pies.

   (a) Ratio: $\frac{4}{6}$ (This is read "________________________".)
   Explanation: This expresses the ratio of the number of pies eaten by John to those eaten by Henry.

   (b) Ratio: __________ (This is read "six over four".)

   Explanation: ____

   ____

   ____

EXERCISE #1

For each of the following, give one ratio. Then explain what is being compared in that ratio.

1. In a bakery there are two ovens. One bakes 20 loaves while the second oven bakes 30 loaves.

   Ratio: __________ Explanation: ________________________

   ____

2. Chuck and Rick are papering a room. For each 3 rolls of wallpaper Rick hangs, Chuck hangs 2 rolls.

   Ratio: ____________ Explanation: ________________________________

   ____

3. Alfred and Jack are learning Morse code. For every 10 letters that Alfred sends on his telegraph key, Jack sends 17.

   Ratio: ____________ Explanation: ________________________________

   ____

4. Tom welds a seam 12 inches long while Bill completes a 29-inch seam.

   Ratio: ____________ Explanation: ________________________________

   ____

5. Rudy and Jerry have been given a pile of resistors to sort. Jerry is able to classify 27 resistors while Rudy classifies 33.

   Ratio: ____________ Explanation: ________________________________

   ____

For each of the following, give the ratio which is specified.

6. George and Sam are carpenters. For every 8 nails George drives, Sam drives 3.

   What is the ratio of the number of nails driven by Sam to those driven by George? ____________

7. At a football game Ron and Tom were opposing quarterbacks. The longest pass completed by Ron was 32 yards while Tom's longest pass completion was 28 yards.

   What is the ratio of the longest pass completed by Ron to that completed by Tom? ____________

8. At one point in a sportscar race, Bruce had completed 29 laps and Mark had completed 30 laps.

   What is the ratio of the number of laps completed by Mark to those completed by Bruce? ____________

Topic Two: Proportion

A. What is a Proportion?

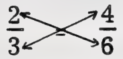
Consider two ratios (fractions) we know to be equivalent. (See Module 2, Lesson 2, page 16.)

$$\frac{2}{3} = \frac{4}{6} \quad \text{since} \quad \left(\frac{2 \times 2}{3 \times 2} = \frac{4}{6}\right)$$

In the next few lessons we will be putting equivalent pairs of ratios to work solving problems. We call these pairs of equivalent ratios "proportions".

> A PROPORTION is an equivalent pair of ratios.

In a proportion, like $\frac{2}{3} = \frac{4}{6}$, we find that the products obtained by multiplying diagonally across the "=" are equal.



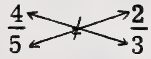
$$\left.\begin{array}{l} 2 \times 6 = 12 \\ 3 \times 4 = 12 \end{array}\right\} \text{We call these } \underline{\text{CROSS}}\ \underline{\text{PRODUCTS}}.$$

Thus for our equivalent ratio $\frac{2}{3} = \frac{4}{6}$, the cross products are equal. (That is, $2 \times 6 = 3 \times 4$ or $3 \times 4 = 2 \times 6$.)

Now let's consider two ratios which are not equivalent. (They do not form a proportion.)

$\frac{4}{5} \neq \frac{2}{3}$ (The symbol "≠" means "is not equal to" or "is not equivalent to".)

We check the cross products.



$4 \times 3 = 12$

$5 \times 2 = 10$

We see that the cross products are not equal. (That is, $4 \times 3 \neq 5 \times 2$ or $5 \times 2 \neq 4 \times 3$.)

The two examples on page 3 prompt the following summary.

Tests for Proportions

(a) If two ratios form a proportion, the cross products are equal.

(b) If the cross products of two ratios are equal, the ratios form a proportion.

(c) If two ratios do not form a proportion, their cross products are not equal.

(d) If the cross products of two ratios are not equal, the ratios do not form a proportion.

EXAMPLES:

Tell whether or not the following pairs of ratios are equivalent.

RATIOS	CROSS PRODUCTS	CONCLUSIONS
(a) $\frac{5}{3}, \frac{7}{4}$	$5 \times 4 = 20$ $3 \times 7 = 21$	These ratios are not equivalent. $\left(\frac{5}{3} \neq \frac{7}{4}\right)$
(b) $\frac{15}{9}, \frac{10}{6}$	$9 \times 10 = 90$ $15 \times 6 = 90$	These ratios are equivalent. $\left(\frac{15}{9} = \frac{10}{6}\right)$

Self-Correcting Exercise #1

1. Place "=" or "≠" between each pair of ratios to specify whether they are equivalent or they are not equivalent.

(a) $\frac{1}{3} \;\underline{\quad\neq\quad}\; \frac{2}{9}$ (b) $\frac{2}{3} \;\underline{\qquad}\; \frac{3}{6}$ (c) $\frac{2}{3} \;\underline{\qquad}\; \frac{20}{30}$

(d) $\frac{1}{3} \;\underline{\qquad}\; \frac{4}{12}$ (e) $\frac{3}{5} \;\underline{\qquad}\; \frac{6}{10}$ (f) $\frac{5}{10} \;\underline{\qquad}\; \frac{50}{100}$

(g) $\frac{1}{3} \;\underline{\qquad}\; \frac{2}{6}$ (h) $\frac{2}{5} \;\underline{\qquad}\; \frac{5}{10}$ (i) $\frac{3}{4} \;\underline{\qquad}\; \frac{75}{100}$

2. From the choices in the right column, circle the ratio which is equivalent to the given ratio.

CALCULATIONS

GIVEN RATIO	CIRCLE ONE
$\frac{2}{6}$	$\frac{1}{2}$, [✓ $\frac{1}{3}$], $\frac{3}{8}$
$\frac{12}{2}$	$\frac{24}{3}$, $\frac{60}{6}$, $\frac{42}{7}$
$\frac{4}{7}$	$\frac{6}{10}$, $\frac{12}{21}$, $\frac{3}{5}$
$\frac{9}{6}$	$\frac{21}{14}$, $\frac{12}{6}$, $\frac{14}{4}$
$\frac{40}{15}$	$\frac{56}{21}$, $\frac{24}{10}$, $\frac{24}{7}$
$\frac{1}{11}$	$\frac{8}{87}$, $\frac{13}{143}$, $\frac{17}{165}$
$\frac{2}{3}$	$\frac{7}{10}$, $\frac{14}{21}$, $\frac{8}{9}$
$\frac{3}{15}$	$\frac{7}{34}$, $\frac{9}{54}$, $\frac{8}{40}$
$\frac{8}{6}$	$\frac{44}{32}$, $\frac{60}{44}$, $\frac{24}{18}$
$\frac{7}{10}$	$\frac{60}{81}$, $\frac{63}{89}$, $\frac{42}{60}$

$\frac{2}{6}$, $\frac{1}{2}$
$2 \times 2 = 4$
$6 \times 1 = 6$
Not equal

$\frac{2}{6}$, $\frac{1}{3}$
$2 \times 3 = 6$
$6 \times 1 = 6$
Equal

The answers to this exercise should be checked with the solutions on page 21.

EXERCISE #2

For each pair of ratios, calculate the cross products. Then put "=" between equivalent ratios or put "≠" between ratios which are not equivalent. (Remember, the symbol "≠" means "is not equivalent to".)

RATIOS	CROSS PRODUCTS	CONCLUSION
$\frac{2}{7}, \frac{16}{59}$	$2 \times 59 = 118$ $7 \times 16 = 112$ not equal	$\frac{2}{7} \neq \frac{16}{59}$
$\frac{3}{4}, \frac{0.15}{0.20}$		
$\frac{4}{5}, \frac{16}{50}$		
$\frac{2}{3}, \frac{10}{15}$		
$\frac{5}{9}, \frac{10}{18}$		
$\frac{15}{3}, \frac{60}{12}$		

B. Finding the Missing Number in a Proportion

There are times when we are given a proportion with one part missing. It is then necessary to find the missing number using the information which is given. We use a variable to hold the place of the missing number.

A VARIABLE is a symbol used to hold the position of an unknown number. Small letters such as "x", "y", "t", "r", and "z" are examples of symbols which are commonly used as variables.

Let's see one way in which these variables can be used.

EXAMPLE:

Find the number which is missing in the proportion

$$\frac{21}{14} = \frac{n}{6}.$$

"n" is the variable used to replace the missing number.

Solution

We use cross products and division to find the value for n.

$$\frac{21}{14} = \frac{n}{6}$$

$14 \times n = 21 \times 6$ (since in any proportion, the cross products are equal.)

Now we divide both sides by the number which multiplies the variable. (In this case, we divide both sides by 14.)

$$\frac{\overset{1}{\cancel{14}} \times n}{\underset{1}{\cancel{14}}} = \frac{\overset{3}{\cancel{21}} \times \overset{3}{\cancel{6}}}{\underset{\underset{1}{\cancel{2}}}{\cancel{14}}} \longleftarrow$$

We cancel factors common to the numerator and denominator of each fraction.

$1 \times n = 9$

$\boxed{n = 9}$

$1 \times n = n$, since 1 times any number leaves the number unchanged.

Once we have found the missing number, we must check to see if it is the correct number. (Sometimes arithmetic errors can result in an incorrect answer.)

Check

$$\frac{21}{14} = \frac{\textcircled{9}}{6}$$

We put the value for "n" in the place which "n" held in the proportion.

$14 \times 9 = 126$

$21 \times 6 = 126$

They are the same!

Since the cross products are equal, 9 is the number for which we were looking.

Here is an additional example of finding the missing number in a proportion. (CAUTION: In this example, the method is shown exactly as you should use it. Every step is needed for careful evaluation.)

EXAMPLE:

Find the value of the variable in the following proportion. Then check your answer.

$$\frac{22}{b} = \frac{24}{60}$$

Solution

$$\frac{22}{b} = \frac{24}{60}$$

$$22 \times 60 = b \times 24 \longleftarrow \text{(The cross products are equal.)}$$

$$\frac{\overset{11}{\cancel{22}} \times \overset{5}{\cancel{60}}}{\underset{\underset{1}{\cancel{2}}}{\cancel{24}}} = \frac{b \times \overset{1}{\cancel{24}}}{\underset{1}{\cancel{24}}} \longleftarrow \text{(Both sides are divided by 24.)}$$

$$\boxed{55 = b}$$

Check

$$\frac{22}{\textcircled{55}} = \frac{24}{60}$$

$$\left.\begin{array}{l} 22 \times 60 = 1320 \leftarrow \\ 55 \times 24 = 1320 \leftarrow \end{array}\right\rangle \text{They are equal.}$$

Calculations

$$\begin{array}{r} 22 \\ \times 60 \\ \hline 1320 \end{array} \qquad \begin{array}{r} 55 \\ \times 24 \\ \hline 220 \\ 110 \\ \hline 1320 \end{array}$$

Thus, b = 55 is the correct solution.

EXERCISE #3

In each case find the missing number, then check your answer. Please show all steps.

1. $\frac{r}{77} = \frac{45}{105}$

$$r \times 105 = 77 \times 45$$

$$\frac{r \times \overset{1}{\cancel{105}}}{\underset{1}{\cancel{105}}} = \frac{\overset{11}{\cancel{77}} \times \overset{3}{\cancel{45}}}{\underset{\underset{1}{\cancel{7}}}{\cancel{105}}}$$

$$r = 33$$

Check

$$\frac{33}{77} = \frac{45}{105}$$

$$\left.\begin{array}{l} 33 \times 105 = 3465 \\ 77 \times 45 = 3465 \end{array}\right\} \text{equal}$$

Calculations

$$\begin{array}{r} 105 \\ \times 33 \\ \hline 315 \\ 315 \\ \hline 3465 \end{array} \qquad \begin{array}{r} 45 \\ \times 77 \\ \hline 315 \\ 315 \\ \hline 3465 \end{array}$$

2. $\frac{16}{32} = \frac{e}{14}$ Check Calculations

3. $\frac{15}{36} = \frac{t}{84}$ Check Calculations

4. $\frac{26}{t} = \frac{39}{45}$ Check Calculations

5. $\frac{e}{42} = \frac{42}{49}$ Check Calculations

6. $\frac{15}{75} = \frac{12}{q}$

Check

Calculations

7. $\frac{70}{n} = \frac{80}{56}$

Check

Calculations

8. $\frac{56}{35} = \frac{64}{i}$

Check

Calculations

Answers to problems 3, 6, and 7 should be checked with the solutions on page 22.

In the next few lessons, we will encounter several problem situations in which we use proportions to find the unknown quantities. The method used in Exercise #3 is a very important tool.

Now we will learn about percents.

Topic Three: Percents

A. Definition of Percent

PERCENT means "per hundred". A percent is a ratio with a denominator of 100. We commonly use the symbol "%" to mean "with a denominator of 100". We refer to "%" as the percent symbol.

EXAMPLES:

$\frac{15}{100}$ can also be written 15%.

(We read "15%" as "fifteen percent".)

275% (two hundred seventy-five percent) is the same as $\frac{275}{100}$.

$\frac{36}{100}$ can also be represented as 36% (thirty-six percent).

100% (one hundred percent) is the same as $\frac{100}{100}$.

By studying the above examples, we see that:

1. A ratio whose value is less than 1 is represented by a percent which is less than 100%.
2. A ratio whose value is 1 is represented by 100%.
3. A ratio whose value is greater than 1 is represented by a percent which is greater than 100%.

Self-Correcting Exercise #2

Change the following percents from ratio form to % form or vice versa.

1. $\frac{39}{100}$ =
2. 16% =
3. 75% =
4. $\frac{125}{100}$ =
5. 253% =
6. $\frac{43}{100}$ =

The answers to this exercise should be checked with the solutions on page 22.

B. Changing a Percent to Simplest Fractional Form

To change a percent to simplest fractional form, we:

1. Write the percent in ratio form. (We now have a fraction.)
2. "Divide out" the largest factor which is common to both terms of the fraction. (See Module 2, Lesson 2 page 26.)

EXAMPLES:

Express the following percents as fractions in simplest form.

(a) 12.5% (b) 9% (c) 24%

Solution

We multiply both numerator and denominator by 2 to clear the decimal from the numerator.

(a) $12.5\% = \frac{12.5}{100} = \frac{12.5 \times 2}{100 \times 2} = \frac{25}{200} = \frac{25 \div 25}{200 \div 25} = \boxed{\frac{1}{8}}$

(b) $9\% = \boxed{\frac{9}{100}} \leftarrow$ (This is in simplest form as there are no common factors in the numerator and denominator.)

(c) $24\% = \frac{24}{100} = \frac{24 \div 4}{100 \div 4} = \boxed{\frac{6}{25}}$

EXERCISE #4

Complete the following table.

%	RATIO	SIMPLEST FORM
6%	$\frac{6}{100}$	$\frac{3}{50}$
20%		
	$\frac{67}{100}$	
125%		

%	RATIO	SIMPLEST FORM
98%		
17%		
43%		
	$\frac{32}{100}$	
70%		
35%		
28%		

C. Changing a Percent to a Decimal Number

EXAMPLE:
Change 56% to a decimal number.

Solution

$56\% = \frac{56}{100} \longleftarrow$ We can write the percent in ratio form.

$\frac{56}{100} = 0.56 \longleftarrow$ (See Module 3, Lesson 2, page 15.)

Thus we have found that 56% = 0.56.

We can summarize the above procedure as follows:

To change a percent to a decimal number, we:

1. Write the percent in ratio form (with a denominator of 100).
2. Change this ratio to a decimal following the rule for dividing by 100 presented in Module 3, Lesson 2, page 15.

EXAMPLES:

(a) $27\% = \frac{27}{100} = 0.27$

(b) $13.5\% = \frac{13.5}{100} = \frac{135}{1000} = 0.135$

(c) $4\% = \frac{4}{100} = 0.04$

(We must use a zero placeholder. Otherwise, we cannot move the decimal point 2 places to the left.)

(d) $298\% = \frac{298}{100} = 2.98$

Self-Correcting Exercise #3

For each of the following, change the percent to a decimal number.

1. $8\% = \frac{\_\_\_}{100} = \_\_\_\_\_\_$

2. $14.2\% =$

3. $19\% =$

4. $123\% =$

5. $36\% =$

6. $32.5\% =$

The answers to this exercise should be checked with the solutions on page 22.

D. Changing a Decimal Number to a Percent

EXAMPLE:

Change 0.613 to a percent.

Solution

$$0.613 = \frac{613}{1000}$$

(Denominator is 1000 since the 3 is in the thousandths place.)

$$= \frac{613 \div 10}{1000 \div 10}$$

(We divide both terms by 10 so we will have a denominator of 100.)

$$= \frac{61.3}{100}$$

$$= 61.3\%$$

When doing problems of this type, we do not normally show all of these steps. We shorten the method to the form in the box below.

> To change a decimal number to a percent, we:
>
> 1. Move the decimal point two places to the right and give the number a denominator of 100.
>
> 2. Rewrite the ratio in % form.

EXAMPLES:

(a) $0.625 = \dfrac{62.5}{100} = 62.5\%$

(b) $0.003 = \dfrac{0.3}{100} = 0.3\%$

(c) $1.2 = \dfrac{120}{100} = 120\%$ (a zero placeholder is used, so we can move the decimal two places to the right.)

(d) $0.32 = \dfrac{32}{100} = 32\%$

Self-Correcting Exercise #4

Write the following decimal numbers in % form.

1. $0.34 = \dfrac{}{100} =$ ______

2. $0.5 =$

3. $0.275 =$

4. $1.76 =$

5. $6.73 =$

6. $0.29 =$

The answers to this exercise should be checked with the solutions on page 23.

EXERCISE #5

Complete the following table:

%	RATIO WITH DENOMINATOR OF 100	DECIMAL NUMBER
5%	$\frac{5}{100}$	0.05
		0.33
58%		
1%		
		1.37
63.5%		
		0.83
		0.432
250%		
0.2%		
7.9%		
		6.92
44%		
		0.7
4.7%		

We have learned how to change a percent into a decimal number and vice versa. We have also learned how to change a percent into a fraction, but we have not learned to change a fraction to a percent.

E. Changing a Fraction to a Percent

We have 2 possible choices of methods for changing a fraction into a percent.

EXAMPLE:

Change $\frac{3}{8}$ into a percent.

Solution

METHOD 1. (PROPORTION)	METHOD 2: (FRACTION TO DECIMAL TO PERCENT)
$\frac{3}{8} = \frac{?}{100}$	
(1) If we can find the numerator when the denominator is 100, then we have the percent. We can use a proportion with a variable to find the numerator. $\frac{3}{8} = \frac{n}{100}$ $8 \times n = 3 \times 100$ $\frac{\overset{1}{\cancel{8}} \times n}{\underset{1}{\cancel{8}}} = \frac{3 \times \overset{25}{\cancel{100}}}{\underset{2}{\cancel{8}}}$ $n = \frac{75}{2}$ $n = 37.5$	(1) We change $\frac{3}{8}$ to the equivalent decimal number by dividing the denominator, 8, into the numerator, 3. $\begin{array}{r} .375 \\ 8\overline{)\,3.000} \\ \underline{2\;4}^{\times\times} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \end{array}$ So $\frac{3}{8} = 0.375$
(2) From the proportion, we have found the percent. $\frac{3}{8} = \frac{37.5}{100}$ So $\frac{3}{8} = 37.5\%$	(2) Change the decimal number to a percent. $0.375 = \frac{37.5}{100} = 37.5\%$ Thus $\frac{3}{8} = 37.5\%$

Thus we summarize the two methods as follows:

> To change a fraction to a percent, we:
>
> 1. use a proportion with a variable for the unknown percent.
>
> (OR)
>
> 2. change the fraction to a decimal number. Then change the decimal number to a percent.

Which method should you choose? Choose the method which is easier for you.

EXAMPLES:

Change the following fractions to percents. Round answers to one decimal place, when necessary.

(a) $\frac{12}{25} = \underline{\quad ? \quad}\%$ (b) $\frac{8}{9} = \underline{\quad ? \quad}\%$

Solutions

(a) $\frac{12}{25} = \underline{\quad 48 \quad}\%$

Method 1	Method 2
(1) $\frac{12}{25} = \frac{x}{100}$	(1) $\frac{12}{25} = ?$
$25 \times x = 12 \times 100$	$\begin{array}{r} .48 \\ 25\,\overline{)12.00} \\ 100^{\times\times} \\ \hline 200 \\ 200 \\ \hline \end{array}$
$\frac{\overset{1}{\cancel{25}} \times x}{\underset{1}{\cancel{25}}} = \frac{12 \times \overset{4}{\cancel{100}}}{\underset{1}{\cancel{25}}}$	
$x = 48$	
(2) $\frac{12}{25} = \frac{48}{100}$	(2) $\frac{12}{25} = 0.48$
	$0.48 = \frac{48}{100} = 48\%$
So $\frac{12}{25} = 48\%$	Thus, $\frac{12}{25} = 48\%$

(b) $\frac{8}{9} = \_\_\_\_\_\_\%$ (Fill in the blanks.)

Method 1

(1) $\frac{8}{9} = \frac{t}{100}$

$\_\_\_ \times t = \_\_\_ \times 100$

$\frac{\_\_\_ \times t}{9} = \frac{8 \times 100}{9}$

$t = \frac{800}{9}$

$t = 88.9$ ← We round to the nearer tenth.

(2) $\frac{8}{9} = \frac{\_\_\_}{100}$

Thus $\frac{8}{9} = 88.9\%$

Method 2

(1) $\frac{8}{9} = ?$

$$\begin{array}{r} .8888 \\ 9\,\overline{)8.0000} \\ \underline{7\,2}^{\times\times\times} \\ 80 \\ \underline{72} \\ 80 \\ \underline{72} \\ 80 \\ \underline{72} \\ 8 \end{array}$$

← We carry the division to 4 decimal places so we can round the quotient to 3 decimal places.

(2) $\frac{8}{9} = 0.889$

$\_\_\_\_ = \frac{\_\_\_}{100} = \_\_\_\_\_\_\%$

So $\frac{8}{9} = \_\_\_\_\_\_\%$

EXERCISE #6

Using Method 1 OR Method 2, change each of the following fractions into a percent. (Round answers to one decimal place where necessary.)

1. $\frac{1}{8} = \_\_\_\_\_\_\%$

2. $\frac{3}{4} = \_\_\_\_\_\_\%$

3. $\frac{5}{6}$ = ________%

4. $\frac{2}{3}$ = ________%

5. $\frac{12}{17}$ = ________%

6. $\frac{5}{7}$ = ________%

7. $\frac{17}{14}$ = ________%

8. $\frac{16}{15}$ = ________%

Answers to problems 1, 3, 5 and 7 should be checked with the solutions on page 23.

Solutions to Selected Exercises

Self-Correcting Exercise #1, page 4.

1. (a) $\frac{1}{3} \neq \frac{2}{9}$, since $1 \times 9 = 9$ and $3 \times 2 = 6$. *The cross products are not equal.*

   (b) $\frac{2}{3} \neq \frac{3}{6}$, since $2 \times 6 = 12$ and $3 \times 3 = 9$.

   (c) $\frac{2}{3} = \frac{20}{30}$, since $3 \times 20 = 60$ and $2 \times 30 = 60$.

   (d) $\frac{1}{3} = \frac{4}{12}$, since $1 \times 12 = 12$ and $3 \times 4 = 12$.

   (e) $\frac{3}{5} = \frac{6}{10}$, since $5 \times 6 = 30$ and $3 \times 10 = 30$.

   (f) $\frac{5}{10} = \frac{50}{100}$, since $5 \times 100 = 500$ and $10 \times 50 = 500$.

   (g) $\frac{1}{3} = \frac{2}{6}$, since $3 \times 2 = 6$ and $1 \times 6 = 6$.

   (h) $\frac{2}{5} \neq \frac{5}{10}$, since $2 \times 10 = 20$ and $5 \times 5 = 25$.

   (i) $\frac{3}{4} = \frac{75}{100}$, since $3 \times 100 = 300$ and $4 \times 75 = 300$.

2.

GIVEN RATIO	CIRCLE ONE
$\frac{2}{6}$	$\frac{1}{2}$, [✓ $\frac{1}{3}$], $\frac{3}{8}$
$\frac{12}{2}$	$\frac{24}{3}$, $\frac{60}{6}$, [✓ $\frac{42}{7}$]
$\frac{4}{7}$	$\frac{6}{10}$, [✓ $\frac{12}{21}$], $\frac{3}{5}$
$\frac{9}{6}$	[✓ $\frac{21}{14}$], $\frac{12}{6}$, $\frac{14}{4}$
$\frac{40}{15}$	[✓ $\frac{56}{21}$], $\frac{24}{10}$, $\frac{24}{7}$
$\frac{1}{11}$	$\frac{8}{87}$, [✓ $\frac{13}{143}$], $\frac{17}{165}$

GIVEN RATIO	CIRCLE ONE
$\frac{2}{3}$	$\frac{7}{10}$, [✓ $\frac{14}{21}$], $\frac{8}{9}$
$\frac{3}{15}$	$\frac{7}{34}$, $\frac{9}{54}$, [✓ $\frac{8}{40}$]
$\frac{8}{6}$	$\frac{44}{32}$, $\frac{60}{44}$, [✓ $\frac{24}{18}$]
$\frac{7}{10}$	$\frac{60}{81}$, $\frac{63}{89}$, [✓ $\frac{42}{60}$]

Exercise #3, page 8.

3. $\frac{15}{36} = \frac{t}{84}$

$15 \times 84 = 36 \times t$

$\frac{\overset{5}{\cancel{15}} \times \overset{7}{\cancel{84}}}{\cancel{36}_{\cancel{12}_{1}}} = \frac{\overset{1}{\cancel{36}} \times t}{\cancel{36}_{1}}$

$\boxed{35 = t}$

Check

$\frac{15}{36} = \frac{35}{84}$

$15 \times 84 = 1260$

equal

$36 \times 35 = 1260$

$\begin{array}{r} 84 \\ \times 15 \\ \hline 420 \\ 84 \\ \hline 1260 \end{array}$

$\begin{array}{r} 36 \\ \times 35 \\ \hline 180 \\ 108 \\ \hline 1260 \end{array}$

6. $\frac{15}{75} = \frac{12}{q}$

$15 \times q = 75 \times 12$

$\frac{\overset{1}{\cancel{15}} \times q}{\cancel{15}_{1}} = \frac{\overset{5}{\cancel{75}} \times 12}{\cancel{15}_{1}}$

$\boxed{q = 60}$

Check

$\frac{15}{75} = \frac{12}{60}$

$15 \times 60 = 900$

equal

$75 \times 12 = 900$

7. $\frac{70}{n} = \frac{80}{56}$

$n \times 80 = 70 \times 56$

$\frac{n \times \overset{1}{\cancel{80}}}{\cancel{80}_{1}} = \frac{\overset{7}{\cancel{70}} \times \overset{7}{\cancel{56}}}{\cancel{80}_{\cancel{8}_{1}}}$

$\boxed{n = 49}$

Check

$\frac{70}{49} = \frac{80}{56}$

$49 \times 80 = 3920$

equal

$70 \times 56 = 3920$

Self-Correcting Exercise #2, page 11.

1. $\frac{39}{100} = \underline{39\%}$
2. $16\% = \underline{\frac{16}{100}}$
3. $75\% = \underline{\frac{75}{100}}$
4. $\frac{125}{100} = \underline{125\%}$
5. $253\% = \underline{\frac{253}{100}}$
6. $\frac{43}{100} = \underline{43\%}$

Self-Correcting Exercise #3, page 14.

1. $8\% = \frac{8}{100} = \underline{0.08}$
2. $14.2\% = \frac{14.2}{100} = \underline{0.142}$
3. $19\% = \frac{19}{100} = \underline{0.19}$
4. $123\% = \frac{123}{100} = \underline{1.23}$
5. $36\% = \frac{36}{100} = \underline{0.36}$
6. $32.5\% = \frac{32.5}{100} = \underline{0.325}$

Self-Correcting Exercise #4, page 15.

1. $0.34 = \frac{34}{100} = 34\%$

2. $0.5 = \frac{50}{100} = 50\%$

A zero placeholder is added so the decimal point can be moved 2 places to the right.

3. $0.275 = \frac{27.5}{100} = 27.5\%$

4. $1.76 = \frac{176}{100} = 176\%$

5. $6.73 = \frac{673}{100} = 673\%$

6. $0.29 = \frac{29}{100} = 29\%$

Exercise #6, page 19.

For each exercise, ONLY ONE of the two methods is needed.

1. $\frac{1}{8} = 12.5\%$

Method 1	Method 2
$\frac{1}{8} = \frac{n}{100}$	$\frac{1}{8} = ?$
$8 \times n = 1 \times 100$	$\begin{array}{r} .125 \\ 8\,\overline{)1.000} \\ \underline{8}^{\times\times} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \end{array}$
$\frac{\cancel{8}^{1} \times n}{\cancel{8}_{1}} = \frac{1 \times \cancel{100}^{25}}{\cancel{8}_{2}}$	
$n = \frac{25}{2} = 12.5$	
$\frac{1}{8} = \frac{12.5}{100}$	$0.125 = \frac{12.5}{100} = 12.5\%$
So, $\frac{1}{8} = 12.5\%$	So $\frac{1}{8} = 12.5\%$

3. $\frac{5}{6} = \underline{\ 83.3\ }\%$

Method 1

$$\frac{5}{6} = \frac{n}{100}$$

$$6 \times n = 5 \times 100$$

$$\frac{\cancel{6}^{1} \times n}{\cancel{6}_{1}} = \frac{5 \times \cancel{100}^{50}}{\cancel{6}_{3}}$$

$$n = \frac{250}{3} = 83.3$$

$$\frac{5}{6} = \frac{83.3}{100}$$

So $\frac{5}{6} = 83.3\%$

Method 2

$$\frac{5}{6} = ?$$

$$\begin{array}{r} .8333 \\ 6\ \overline{)5.0000} \\ \underline{4\ 8}^{\times\times\times} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

$$0.833 = \frac{83.3}{100} = 83.3\%$$

So $\frac{5}{6} = 83.3\%$

5. $\frac{12}{17} = \underline{\ 70.6\ }\%$

Method 1

$$\frac{12}{17} = \frac{n}{100}$$

$$12 \times 100 = 17 \times n$$

$$\frac{12 \times 100}{17} = \frac{\cancel{17}^{1} \times n}{\cancel{17}_{1}}$$

$$\frac{1200}{17} = n$$

$$\begin{array}{r} 70.58 \\ 17\ \overline{)1200.00} \\ \underline{119}^{\times\ \times\times} \\ 10 \\ \underline{0} \\ 100 \\ \underline{85} \\ 150 \\ \underline{136} \\ 14 \end{array}$$

$$70.6 = n$$

So $\frac{12}{17} = \frac{70.6}{100} = 70.6\%$

Method 2

$$\frac{12}{17} = ?$$

$$\begin{array}{r} .7058 \\ 17\ \overline{)12.0000} \\ \underline{11\ 9}^{\times\times\times} \\ 10 \\ \underline{0} \\ 100 \\ \underline{85} \\ 150 \\ \underline{136} \\ 14 \end{array}$$

$$0.706 = \frac{70.6}{100} = 70.6\%$$

So $\frac{12}{17} = 70.6\%$

7. $\frac{17}{14} = \underline{\quad 121.4 \quad}\%$

Method 1

$$\frac{17}{14} = \frac{n}{100}$$

$$14 \times n = 17 \times 100$$

$$\frac{\overset{1}{\cancel{14}} \times n}{\underset{1}{\cancel{14}}} = \frac{17 \times \overset{50}{\cancel{100}}}{\underset{7}{\cancel{14}}}$$

$$n = \frac{850}{7}$$

$$\begin{array}{rr}
 & 121.42 \\
7 & \overline{)850.00} \\
 & \underline{7} \\
 & 15 \\
 & \underline{14} \\
 & 10 \\
 & \underline{7} \\
 & 30 \\
 & \underline{28} \\
 & 20 \\
 & \underline{14} \\
 & 6
\end{array}$$

$$n = 121.4$$

$$\frac{17}{14} = \frac{121.4}{100}$$

So $\frac{17}{14} = 121.4\%$

Method 2

$$\frac{17}{14} = ?$$

$$\begin{array}{rr}
 & 1.2142 \\
14 & \overline{)17.0000} \\
 & \underline{14} \\
 & 30 \\
 & \underline{28} \\
 & 20 \\
 & \underline{14} \\
 & 60 \\
 & \underline{56} \\
 & 40 \\
 & \underline{28} \\
 & 12
\end{array}$$

$$1.214 = \frac{121.4}{100} = 121.4\%$$

So $\frac{17}{14} = 121.4\%$

End of Lesson 2

Practical Mathematics
Module 4
Lesson 3

PROBLEMS INVOLVING PERCENTS

In the last lesson, we encountered percents. In this lesson, we are going to see how they can be used to solve problems.

Topic One: Setting up Percentage Problems

In problems involving percents, we have three basic variable elements from which we set up the proportion.

$$\boxed{\frac{a}{b} = \frac{r}{100}}$$

The three variable elements in the proportion are defined as follows:

a (amount) -	This is the number or the quantity which is to be compared. (It is the numerator of the first ratio.)
b (base) -	This is the number we are comparing to. (It is the denominator of the first ratio.)
r (rate) -	This is the number part of the percent. (Together with the denominator of 100, this forms the second ratio.)

Using the above proportion $\left(\frac{a}{b} = \frac{r}{100}\right)$, we are able to find the missing quantity when the other two quantities are given. From this we gather that there are only three types of problems involving percents:

(1) those in which the amount is unknown;

(2) those in which the base is not given; and

(3) those in which the rate is not given.

The missing quantity is found by solving the proportion.

EXAMPLE 1:

(a) What is 25% of \$444?

Solution

We must first identify a, b, and r. In general, we find that all percent problems can be changed into the sentence

$$\boxed{\text{a is r \% of b.}}$$

The base is usually preceded by the word "of."

In this case, we have

$$\$\underset{a}{\underline{\qquad\quad}} \text{ is } \underset{r}{\underline{25}}\% \text{ of } \underset{b}{\underline{\$444}}.$$

We now use this information in the proportion

$$\frac{a}{b} = \frac{r}{100}.$$

Substituting for b and r, we get

$$\frac{a}{444} = \frac{25}{100}.$$

Since the percent is less than 100%, the amount must be less than the base. (i.e, each numerator must be smaller than the corresponding denominator.)

Now solve the proportion for a.

$$a \times 100 = 444 \times 25$$

$$\frac{a \times \cancel{100}^{1}}{\cancel{100}_{1}} = \frac{\overset{111}{\cancel{444}} \times \cancel{25}^{1}}{\cancel{100}_{\cancel{4},1}}$$

$$a = 111 \longleftarrow \text{Note: a is less than b, since \$111 is less than \$444.}$$

Thus \$111 is 25% of \$444.

Now try this one yourself.

(b) What is 23% of 82?

Solution

$$\underset{a}{\underline{\qquad\quad}} \text{ is } \underset{r}{\underline{\qquad\quad}}\% \text{ of } \underset{b}{\underline{\qquad\quad}}.$$

EXAMPLE 2:

(a) What percent of 19.50 is 130?

Solution

$$\underset{a\nearrow}{\underline{130}}\ \text{is}\ \underset{r\nearrow}{\underline{\qquad\quad}}\%\ \text{of}\ \underset{b\nearrow}{\underline{19.50}}$$

$$\frac{130}{19.50} = \frac{r}{100}$$

Since the amount is larger than the base, r will be more than 100%. (That is, each numerator is larger than its corresponding denominator.)

$$19.50 \times r = 130 \times 100$$

$$\frac{\overset{1}{\cancel{19.50}} \times r}{\underset{1}{\cancel{19.50}}} = \frac{\overset{10}{\cancel{130}} \times \cancel{100}^{2}}{\cancel{19.50}_{\cancel{0.39}\ 0.03}}$$

$$r = \frac{20}{0.03} = \frac{2000}{3}$$

$$r = 666\frac{2}{3}$$

Note: r is greater than 100, since $666\frac{2}{3}$ is greater than 100.

So, 130 is $666\frac{2}{3}$% of 19.50.

Now try this one on your own.

(b) What percent of 416 is 520?

Solution

$$\underset{a\nearrow}{\underline{\qquad\quad}}\ \text{is}\ \underset{r\nearrow}{\underline{\qquad\quad}}\%\ \text{of}\ \underset{b\nearrow}{\underline{\qquad\quad}}.$$

EXAMPLE 3:

(a) $113\frac{3}{4}$ is $62\frac{1}{2}$% of what number?

Solution

$$\underset{a\nearrow}{\underline{113\tfrac{3}{4}}} \text{ is } \underset{r\nearrow}{\underline{62\tfrac{1}{2}}}\% \text{ of } \underset{b\nearrow}{\underline{\qquad\quad}}.$$

$$\frac{113\frac{3}{4}}{b} = \frac{62\frac{1}{2}}{100}$$

$$b \times 62\frac{1}{2} = 113\frac{3}{4} \times 100$$

We must change the mixed numbers into improper fractions before multiplying or dividing.

$$\frac{b \times \frac{\overset{1}{\cancel{125}}}{\cancel{2}}}{\frac{\cancel{125}}{\cancel{2}_{1}}} = \frac{\frac{455}{4} \times 100}{\frac{125}{2}}$$

$$b = \frac{\frac{455 \times 100}{4}}{\frac{125}{2}}$$

$$b = \frac{455 \times 100}{4} \div \frac{125}{2}$$

$$b = \frac{\overset{91}{\cancel{455}} \times \overset{\overset{1}{\cancel{25}}}{\cancel{100}}}{\cancel{4}_{1}} \times \frac{2}{\cancel{125}_{\cancel{5}_{1}}} \leftarrow$$

We multiply by the reciprocal of the divisor!

$$b = 182$$

Thus $113\frac{3}{4}$ is $62\frac{1}{2}\%$ of 182.

Try your skill at the next problem.

(b) 6% of what number is 15?

Solution

$$\underset{a\nearrow}{\underline{\qquad\quad}} \text{ is } \underset{r\nearrow}{\underline{\qquad\quad}}\% \text{ of } \underset{b\nearrow}{\underline{\qquad\quad}}.$$

Self-Correcting Exercise #1

1. For each of the following, put the problem into the general sentence form "a is r% of b". (Leave a blank for the unknown quantity.)

   (a) What is 5% of 93?

   ______ is 5% of 93.

   (b) 71% of what number is 152?

   (c) 34 is what percent of 19?

   (d) 23% of 68 is what number?

   (e) 87% of what number is 929?

2. Using each of the following problems expressed in the general sentence form "a is r% of b", set up the ratio $\frac{a}{b} = \frac{r}{100}$.

   (a) 63 is r% of 29.

   $\frac{63}{29} = \frac{r}{100}$

   (b) a is 17% of 138.

   (c) 44 is 32% of b.

   (d) a is 296% of 137.

   (e) 51 is r% of 17.

3. Solve each of the following problems using the same method shown in EXAMPLES #1, 2 and 3. (As the first step in the solution, put the problem into the form "a is r% of b".)

   (a) What percent of \$5000 is \$17.50?

   (b) What is $2\frac{1}{2}$% of \$1297?

   (c) 525 is 16% of what number?

Answers to this exercise should be checked against the solutions on page 24.

EXERCISE #1

Solve the following problems using the same method shown in EXAMPLES #1, 2, and 3.

1. What is $3\frac{1}{4}\%$ of $548.60?

2. $45,414 is 120% of what number?

3. 13 is what percent of 52?

4. 92 is what percent of 12?

5. Find 375% of 96 metres. (Remember, the amount must be greater than 96 metres, because the rate is greater than 100%.)

Topic Two: More Problems Involving Percents

We can use our method for solving percentage problems to deal with more complex problems. The general principles are the same as in EXERCISE #1. The procedure to use is summarized here:

Problem-Solving Method

1. Carefully read through the problem.
2. Analyze the problem — put it into the general sentence form (a is r% of b) to identify the base, rate, and amount.
3. Use the proportion $\frac{a}{b} = \frac{r}{100}$ to solve for the unknown element.
4. Use a sentence to relate the answer to the question.

EXAMPLES:

(a) 4" of tubing is cut from a 20" length of tubing. What percent of the tubing is left? (Hint: First find the length of tubing which is left.)

Solution

Amount of tubing left: 20" - 4" = 16"

$\underset{a\nearrow}{\underline{16''}}$ is $\underset{r\nearrow}{\underline{\qquad\qquad}}$% of $\underset{b\nearrow}{\underline{20''}}$

(The base is usually the original quantity to which we make the comparison.)

$$\frac{16}{20} = \frac{r}{100}$$

$$20 \times r = 16 \times 100$$

$$\frac{\overset{1}{\cancel{20}} \times r}{\underset{1}{\cancel{20}}} = \frac{16 \times \overset{5}{\cancel{100}}}{\underset{1}{\cancel{20}}}$$

$$r = 80$$

80% of the tubing is left.

(b) Acme Timber Products experienced a substantial increase in sales in 1973. The 1973 sales were 150% of the 1972 sales. If the 1972 sales were \$174,500, what were the 1973 sales?

The underlined sentence is in the form of the general sentence "a is r % of b."

Solution

$$\underset{a}{\$\underline{\qquad}} \text{ is } \underset{r}{\underline{150\%}} \text{ of } \underset{b}{\underline{\$174,500}}.$$

$$\frac{a}{174500} = \frac{150}{100}$$

Since the rate is more than 100%, we expect the amount to be larger than the base.

$$a \times 100 = 174500 \times 150$$

$$\frac{a \times \cancelto{1}{100}}{\cancelto{1}{100}} = \frac{\cancelto{1745}{174500} \times 150}{\cancelto{1}{100}}$$

$$a = 261,750$$

Note: the amount is indeed greater than the base.

The 1973 sales were \$261,750.

(c) Ron Green paid \$2368.50 income tax last year. The rate at which he paid was 20% of his gross income. How much did he earn last year?

"of" indicates that his gross income is the base.

Solution

$$\underset{a}{\$\underline{2368.50}} \text{ is } \underset{r}{\underline{20\%}} \text{ of } \underset{b}{\$\underline{\qquad}}.$$

$$\frac{2368.50}{b} = \frac{20}{100}$$

$$b \times 20 = 2368.50 \times 100$$

$$\frac{b \times \cancelto{1}{20}}{\cancelto{1}{20}} = \frac{2368.50 \times \cancelto{5}{100}}{\cancelto{1}{20}}$$

$$b = 11,842.50$$

Ron's gross income was \$11,842.50.

Self-Correcting Exercise #2

For each of the following problems, write the general sentence "a is r% of b." (No solution is necessary.)

1. In British Columbia, the sales tax is 5% of the selling price. If a color television sells for $550.00, how much sales tax must be paid?

2. The Tow and Go ski shop was having their annual spring sale. If the discount was 20%, how much could Gary expect to save on a $375 pair of skis?

3. After 3 hours of work, the painters had painted 4300 square feet of a job in which 10,350 square feet must be painted. What percent of the job was completed after 3 hours?

4. Dave saved 10% of his income. If Dave was able to save $832.50 each year, what was his income?

Answers to this exercise should be checked against the Solutions on page 25.

EXERCISE #2

Solve the following problems.

1. Solder is a mixture of tin and lead. In soft solder, 38% of the total weight is tin and in hard solder, 65% of the total weight is tin. Find the amount of tin and lead in (a) a 5 kg bar of hard solder, and (b) a 10 kg roll of soft solder.

(a) Solution ① amount of tin

$\underset{a\nearrow}{\underline{\qquad}}$ kg is $\underset{r\nearrow}{\underline{65}}$% of $\underset{b\nearrow}{\underline{5}}$ kg

$$\frac{a}{5} = \frac{65}{100}$$

$$a \times 100 = 5 \times 65$$

$$\frac{a \times \cancel{100}^{1}}{\cancel{100}_{1}} = \frac{\cancel{5}^{1} \times \cancel{65}^{13}}{\cancel{100}\,\cancel{20}_{4}}$$

$$a = \frac{13}{4} \quad \text{or} \quad 3.25$$

There are 3.25 kg of tin in a 5-kg bar of hard solder.

② amount of lead

Since the mixture is only tin and lead,

Total weight − amount of tin = Amount of lead

________ − __________ = __________

Thus, in a 5-kg bar of hard solder, there are 3.25 kg of tin and ____ kg of lead.

(b) Solution ① amount of tin in 10 kg of soft solder

② amount of lead in 10 kg of soft solder

2. Eugene is an electrician. He earns $550 per week. 30% of his salary is taken off his check for income tax, union dues, and other deductions.
   (a) What amount is deducted from his check each week?
   (b) What is his take-home pay?

3. A brass casting is composed of 30 kg copper, 1 kg tin, 2 kg lead, and 13 kg zinc. (a) Find the total weight of the casting. (b) What percent of the total weight is zinc? (c) What percent of the total weight is copper? (Round the rates in (b) and (c) to the nearer percent.)

4. A carpenter uses 420 board feet of lumber on a job. This represents 85% of the lumber delivered to the site. How many board feet of lumber were delivered? (Round answer to the nearer board foot.)

5. A belt transmits 45.5 horsepower to an engine. This is only 65% of the power input to the belt. (a) What is the power input? (b) What power has been lost in the transmission?

6. On an exam, Paul had 13 problems incorrect. If there were 80 problems on the exam, what was his mark in percentage? (Hint: You must first find how many problems he had correct. Then calculate what percent of the total number of problems he had correct.) Round answer to nearer percent.

7. An engine uses 1,600 pounds of a mixture of fuel and air. If $12\frac{1}{2}\%$ of this mixture is fuel, how many pounds of fuel are used?

8. A pipefitter uses 27.3% of his stock of pipe. If his total stock is 320 m, how many metres has he used? (Round answer to the nearer metre.)

9. A machinist earns $8.70 per hour, with time and one half for overtime. If he works 44 hours straight time and 12 hours overtime, what percent of his total pay is his overtime pay? (Round to the nearer percent.)

10. In a plant of 2,000 employees, 2% were absent in one day. How many employees were absent?

Topic Three: Percent Increase, Percent Decrease, Percent Error and Discounts.

Percent increase, percent decrease, percent error, and discounts are further applications of percentage problems. They have readily identifiable "amounts", but the "bases" are more difficult to identify.

Generally, we will find the following summary useful for identifying the "amount" and the "base".

Percent Increase	Percent Decrease	Percent Error	Discount
a - actual amount of increase	a - actual amount of decrease	a - actual amount of error	a - amount to be marked off
b - original quantity before the increase	b - original quantity before the decrease	b - required or standard quantity	b - original price before the discount is subtracted
r - rate	r - rate	r - rate	r - rate

EXAMPLE 1:

Big John's Lumber Company is preparing a shipment of lumber. The order calls for one 6" × 6" (six by six) timber which is 13.5 feet long. The quality control section measures the timber and finds its length to be 13.2 feet. What percent error is represented by the actual length of the timber?

Solution

Actual amount of error: 13.5 - 13.2 = 0.3 feet
Required measurement: 13.5 feet

$$\underset{a\nearrow}{\underline{0.3}}\text{ feet is }\underset{r\nearrow}{\underline{\qquad}}\%\text{ of }\underset{b\nearrow}{\underline{13.5}}\text{ feet}$$

$$\frac{0.3}{13.5} = \frac{r}{100}$$

$$13.5 \times r = 0.3 \times 100$$

$$\frac{\overset{1}{\cancel{13.5}} \times r}{\underset{1}{\cancel{13.5}}} = \frac{\overset{0.1}{\cancel{0.3}} \times \overset{20}{\cancel{100}}}{\cancel{13.5}_{\cancel{4.5}_{0.9}}}$$

$$r = \frac{2}{0.9} = \frac{20}{9}$$

$$r = 2.2$$

The timber had a 2.2% error from the specified 13.5 feet.

EXAMPLE 2:

In the International Union of Operating Engineers, a journeyman welder received a 6.2% increase in pay on April 1, 1978. (a) If his hourly wage was \$8.10 before the increase, what is the hourly increase he received? (Round the answer to the nearer cent.) (b) What was his new hourly wage?

Solution

(a) The amount of increase is not given, but the base pay is.

$$\underset{a}{\underline{\qquad}} \text{ is } \underset{r}{\underline{6.2\%}} \text{ of } \underset{b}{\underline{\$8.10}}$$

$$\frac{a}{8.10} = \frac{6.2}{100}$$

← COMPLETE

Thus, the welder received a ________ per hour increase in pay.

(b) New Wage = Old Wage + Increase

= ________ + ________

= ________

His new wage was ________.

EXAMPLE 3:

In Central Alberta a workman noted that on December 22 (the shortest day) the sun rose at 8:45 a.m. and set at 4:15 p.m. On June 22 (the longest day) the sun rose at 5:00 a.m. and set at 10:00 p.m. What is the percent increase in the number of hours of sunlight on those dates?

Solution

First we must find the number of hours of sunlight on each of the dates.

	Morning Hours	Afternoon Hours	TOTAL
Shortest Day	There are 3 hours 15 minutes from 8:45 a.m. to 12 noon. 3 hr. 15 min. = $3\frac{15}{60}$ $= 3\frac{1}{4}$ = 3.25 hr.	There are 4 hours 15 minutes from 12 noon to 4:15 p.m. 4 hr. 15 min. = $4\frac{15}{60}$ $= 4\frac{1}{4}$ = 4.25 hr.	3.25 + 4.25 = 7.50 hr.
Longest Day	There are 7 hours from 5 a.m. to 12 noon.	There are 10 hours from 12 noon to 10 p.m.	7 + 10 = 17 hr.

Amount of increase: 17 - 7.50 = 9.50 hr.

Base (or shortest day): 7.50 hr.

$\underset{a}{\underline{9.50}}$ hr. is $\underset{r}{\underline{\qquad}}$% of $\underset{b}{\underline{7.50}}$ hr.

$$\frac{9.50}{7.50} = \frac{r}{100}$$

Since the numerator is more than the denominator, r will be more than 100.

← COMPLETE

Thus the increase in the number of hours of sunlight was __________% (to the nearer percent).

EXAMPLE 4:

Tony purchased a 5 metre cedar canoe on sale for \$884. If the original price was \$1040, what percent discount did he receive?

Solution

Amount of discount: \$1040 - \$884 = \$156
Base or original price: \$1040

$\underset{a \nearrow}{\$1\underline{56}}$ is $\underset{r \nearrow}{\underline{\qquad\quad}}$% of $\underset{b \nearrow}{\$10\underline{40}}$

$$\frac{156}{1040} = \frac{r}{100}$$

← COMPLETE

Tony received a _______% discount.

EXERCISE #3

1. A car which costs \$5375 depreciates 25% during the first year. (a) How much is lost to depreciation during the first year? (b) What is the value of the car after the first year?

2. The selling price of a house and lot have increased by 450% over a period of 20 years. (a) By what amount has the price increased, if the original price of the house and lot was $12,500? (b) What is the present value of this property?

3. A plank has a measurement which is 0.5 dm in error. (a) If this represents a 3% error, find the standard measurement to the nearer tenth of a dm. (b) If the plank is shorter than the standard measurement, how long is it?

4. A circuit loses 5 volts in the wiring resistance. This loss is $3\frac{1}{2}\%$ of the line voltage. What is the line voltage to the nearer volt?

5. An overnight camping trip was being planned for a troop of Boy Scouts. 15 two-man tents were being purchased for the outing. Each tent cost \$49.98 and the troop was given an 18% discount on the purchase.
   (a) What was the total cost of the 15 tents before the discount?
   (b) What was the amount of the discount taken off the total price?
   (c) What was the total amount paid by the Boy Scout troop?

Topic Four: Interest

A knowledge of percents is also necessary when we must calculate interest.

INTEREST is a fee paid for the use of money loaned or borrowed.

Interest is usually calculated on a percentage basis. In calculating interest, you will use a special interest formula rather than the proportion $\frac{a}{b} = \frac{r}{100}$ you used in other percentage problems.

INTEREST FORMULA

$$\underline{i = p \times r \times t}$$

where i is the interest;
p is the principal (ie, the amount borrowed or the amount deposited);
r is the rate (in ratio or decimal form);
and t is the time for which the money is borrowed or deposited. (This must correspond to the time period quoted in the rate).

EXAMPLE 1:

William Smithers borrowed $8,000 from the bank at a rate of $12\frac{1}{4}\%$ per year. After two years, Mr. Smithers returned to the bank to pay off the entire loan plus the interest. How much did he pay?

Solution

$$\underset{\text{(We are calculating the interest)}}{i = \underline{\qquad}}, \quad \underset{\text{(the amount borrowed)}}{p = \$8{,}000}, \quad r = 12\tfrac{1}{4}\% \xrightarrow{\text{Percent}} 12.25\% = \overset{\text{decimal form}}{0.1225}, \quad \underset{\text{(Since the rate is expressed as a yearly rate, the time must be given in years.)}}{t = 2 \text{ yrs.}}$$

$$i = p \times r \times t$$
$$i = 8{,}000 \times 0.1225 \times 2$$
$$i = \$1960$$

Thus, he will pay $1960 in interest.

We must now calculate the total amount he paid the bank (ie, principal plus interest).

Total Amount = Principal + Interest

= ________ + ________

= ________

Mr. Smithers paid the bank __________ in repayment of his $8,000 loan.

EXAMPLE 2:

What is the interest paid on a $1500 loan, if the interest rate is 9% per year and the term of the loan is 30 months?

Solution

$i = \_\_\_\_\_\_, \quad p = \$1500, \quad r = 9\% = 0.09, \quad t = 30 \text{ mo.} = \frac{5}{2} \text{ yr.}$

$$i = p \times r \times t$$
$$i = 1500 \times 0.09 \times \frac{5}{2}$$
$$i = \$337.50$$

(*Since the rate is a yearly rate, the time must be expressed in years.* [$30 \text{ mo.} = \frac{30}{12} \text{ yr.} = \frac{5}{2} \text{ yr.}$])

Thus, $337.50 in interest must be paid on the loan.

EXAMPLE 3:

Charlie Jones made a $110 payment on his CHARGEX bill of $432. He must pay $1\frac{1}{2}\%$ per month on the unpaid balance. How much interest will be due on Charlie's next bill?

Solution

$i = \_\_\_\_\_\_, \quad p \text{ (unpaid balance)} = \$432 - \_\_\_ = \_\_\_, \quad r = 1\frac{1}{2}\% = 1.5\% = 0.015,$

$t = 1 \text{ month}$

(*Since the rate is "per month," the time must be expressed in months.*)

$$i = p \times r \times t$$
$$i = \_\_\_\_\_\_ \times 0.015 \times 1$$
$$i = \_\_\_\_\_\_$$

On next month's bill, Charlie will have __________ interest due.

EXERCISE #4

1. Jack borrowed $1,600 from the bank at a rate of 11% per year for 3 months. How much interest will he owe at the end of the 3 months?

2. Robert Blackford invested $13,000 in a term certificate at $9\frac{1}{2}$% per year. The term of the certificate was 3 years. How much money will Robert have at the end of the three years?

3. On his vacation, George used a credit card to make several purchases. He charged a total of $835. When the bill came due, he was only able to make a payment of $35. If interest is charged at a rate of $1\frac{1}{2}$% per month on the unpaid balance, how much interest will he have to pay next month?

4. To buy a new car, Ralph borrowed \$2,000 from his Uncle Paul. At the end of 18 months, how much money did Ralph owe Paul? (The agreed rate of interest was $8\frac{3}{4}$% per year.)

Solutions to Self-Correcting Exercises

Self-Correcting Exercise #1, page 5

1. (b) 152 is 71% of ______.

   (c) 34 is ______% of 19.

   (d) ______ is 23% of 68.

   (e) 929 is 87% of ______.

2. (b) $\frac{a}{138} = \frac{17}{100}$ (c) $\frac{44}{b} = \frac{32}{100}$

   (d) $\frac{a}{137} = \frac{296}{100}$ (e) $\frac{51}{17} = \frac{r}{100}$

3. (a) \$17.50 is $\underset{r \nearrow}{\underline{\hspace{2cm}}}$% of \$5000

$$\frac{17.50}{5000} = \frac{r}{100}$$

$$5000 \times r = 17.50 \times 100$$

$$\frac{\overset{1}{\cancel{5000}} \times r}{\underset{1}{\cancel{5000}}} = \frac{\overset{0.35}{\cancel{17.50}} \times \overset{1}{\cancel{100}}}{\underset{\underset{1}{\cancel{50}}}{\cancel{5000}}}$$

$$r = 0.35$$

Thus \$17.50 is 0.35% of \$5000.

(b) $\underset{a \nearrow}{\underline{\qquad\qquad}}$ is $2\frac{1}{2}$% of $1297.

$$\frac{a}{1297} = \frac{2.5}{100} \qquad \left(2\tfrac{1}{2} = 2.5\right)$$

$$a \times 100 = 1297 \times 2.5$$

$$\frac{a \times \cancel{100}^{1}}{\cancel{100}_{1}} = \frac{1297 \times \cancel{2.5}^{0.1}}{\cancel{100}_{4}}$$

$$a = \frac{129.7}{4} = \$32.43 \qquad \text{(to the nearer cent)}$$

```
     32.425
  4)129.700
    12
     09
      8
      17
      16
       10
        8
        20
        20
         0
```

Thus $32.43 is $2\frac{1}{2}$% of $1297.

(c) 525 is 16% of $\underset{b \nearrow}{\underline{\qquad\qquad}}$.

$$\frac{525}{b} = \frac{16}{100}$$

$$b \times 16 = 525 \times 100$$

$$\frac{b \times \cancel{16}^{1}}{\cancel{16}_{1}} = \frac{525 \times \cancel{100}^{25}}{\cancel{16}_{4}}$$

```
   525
 × 25
  2625
 1050
 13125
```

$$b = \frac{13125}{4} = 3281.25$$

```
     3281.25
  4)13125.00
    12
     11
      8
      32
      32
       05
        4
        10
         8
         20
         20
          0
```

Thus 525 is 16% of 3281.25.

<u>Self-Correcting Exercise #2, page 10</u>

1. $\underset{a \nearrow}{\underline{\qquad\qquad}}$ is 5% of $550.
2. $\underset{a \nearrow}{\underline{\qquad\qquad}}$ is 20% of $375.
3. 4300 sq. ft. is $\underset{r \nearrow}{\underline{\qquad\qquad}}$% of 10,350 sq. ft.
4. $832.50 is 10% of $\underset{b \nearrow}{\underline{\qquad\qquad}}$.

End of Lesson 3

Practical Mathematics
Module 4
Lesson 4

VARIATION

In some real-life situations, two variable quantities are related in such a way that a change in one quantity causes a change in the other. For example, each time the price of a litre of milk rises, you must pay more; and you also receive less change from a dollar after paying for it. Thus we see that a variation in the price of milk causes variation both in the amount you must pay and in the change you receive.

We will be considering two types of variation in this course — direct and inverse. First, we consider direct variation.

Topic One: Direct Variation

> When two quantities VARY DIRECTLY, an increase in one quantity causes a certain increase in the other quantity. DIRECT VARIATION also occurs when a decrease in one quantity causes a certain decrease in the other quantity.

The following are examples of direct variation:

1. The number of tomatoes Mary buys varies directly as the total cost. This means that the more tomatoes Mary buys, the more they will cost.

eg., 1 kg $\xrightarrow{\text{costs}}$ 89¢
2 kg $\xrightarrow{\text{cost}}$ \$1.78
3 kg $\xrightarrow{\text{cost}}$ \$2.67

Note that for each increase of 1 kg, the cost always increases by 89¢.

2. The distance travelled by an automobile at a fixed rate of speed varies directly as the travelling time. Thus, the longer a car travels at a certain speed, the farther it goes.

eg., If the speed is 100 km/h
1 hour $\xrightarrow{\text{yields}}$ 100 km
2 hours $\xrightarrow{\text{yield}}$ 200 km
6 hours $\xrightarrow{\text{yield}}$ 300 km

Thus, as the travelling time increases, so does the distance.

Looking back at the previous examples, we find that taking any two pairs of related quantities yields a proportion.

(1)

$$\begin{aligned} &1\text{ kg} \xrightarrow{\text{costs}} 89¢ \\ &2\text{ kg} \xrightarrow{\text{cost}} \$1.78 \end{aligned}$$

$$\left.\frac{1}{2} = \frac{89}{178}\right\} \quad \text{since} \quad 1 \times 178 = 178 \text{ and } 2 \times 89 = 178$$

(2)

$$\begin{aligned} &1\text{ hour} \xrightarrow{\text{yields}} 100\text{ km} \\ &6\text{ hours} \xrightarrow{\text{yield}} 600\text{ km} \end{aligned}$$

$$\left.\frac{1}{6} = \frac{100}{600}\right\} \quad \text{since} \quad 1 \times 600 = 600 \text{ and } 6 \times 100 = 600$$

In any direct variation problem, we can set up a proportion. If any member of the proportion is missing, then we can solve for it as we have done in the previous lessons.

EXAMPLE 1:

The local supermarket had a sale on tomato soup. If the sale price was 2 cans for 69 cents, how much did Alice pay for 12 cans?

Solution

$$\begin{aligned} &2\text{ cans} \xrightarrow{\text{cost}} \$0.69 \\ &12\text{ cans} \xrightarrow{\text{cost}} \$\ \ c \end{aligned}$$

We must find how much the 12 cans cost.

When we set up the proportion, we drop the units from the quantities.

$$\frac{2}{12} = \frac{0.69}{c}$$

Notice that 2 cans and the cost of those 2 cans are both numerators.
Also note that 12 cans and the cost of the 12 cans are both denominators.

$$2 \times c = 12 \times 0.69$$

$$\frac{\cancel{2}^{1} \times c}{\cancel{2}_{1}} = \frac{\cancel{12}^{6} \times 0.69}{\cancel{2}_{1}}$$

$$c = \$4.14$$

Then we reintroduce the units ($) for the answer.

Thus, Alice must pay $4.14 for the 12 cans of soup.

EXAMPLE 2:

A jet takes $8\frac{1}{2}$ hours to travel 8 000 km. At the same average speed, how long will it take to travel 2,100 km?

Solution

$$8\tfrac{1}{2}\text{ hr.} \xrightarrow{\text{yield}} 8\ 000\text{ km}$$

$$y\text{ hr.} \xrightarrow{\text{yield}} 2100\text{ km}$$

$$\frac{8\frac{1}{2}}{y} = \frac{8000}{2100}$$

Remember to change $8\frac{1}{2}$ to an improper fraction or to a decimal number before calculating the answer.

← COMPLETE

It takes __________ hours for the plane to make the 2100 km trip.

In our examples, we notice that in direct variation, the two related quantities (ie., 2 cans for 69 cents) are in both numerators or in both denominators of the proportion.

Self-Correcting Exercise #1

In each of the following direct variation problems, set up a proportion. Use the variable n for the missing quantity.

1. In the Danish Bakery, 1 dozen donuts costs 85¢. How much do 6 donuts cost?

2. If 1000 ball bearings weigh 300 kg, how much will 3425 ball bearings weigh? (All are approximately the same size and weight.)

3. John travelled 320 km in 4 hours on his motorcycle. How far will he have travelled after 7 hours, if he maintains the same average speed?

Answers to this exercise should be checked with the solutions on page 20.

EXERCISE #1

Solve the following direct variation problems. (You may refer back to EXAMPLES #1 and 2.)

1. If 100 metal washers weigh $\frac{5}{8}$ kg, how many washers weigh 40 kg?

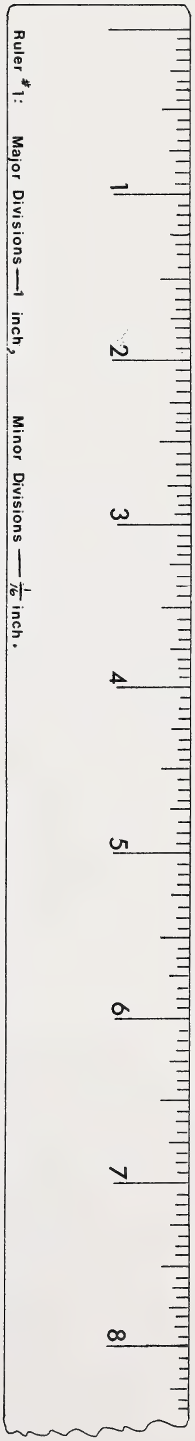
2. A shaft tapers 0.008 cm in 6 cm. What is the length of a piece which has a taper of 0.048 cm?

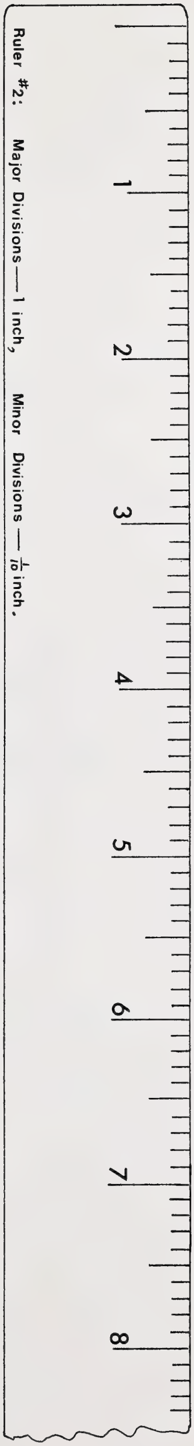
3. If a car runs 51 km on $1\frac{1}{2}$ litres of gas, how far will it run on 15 litres?

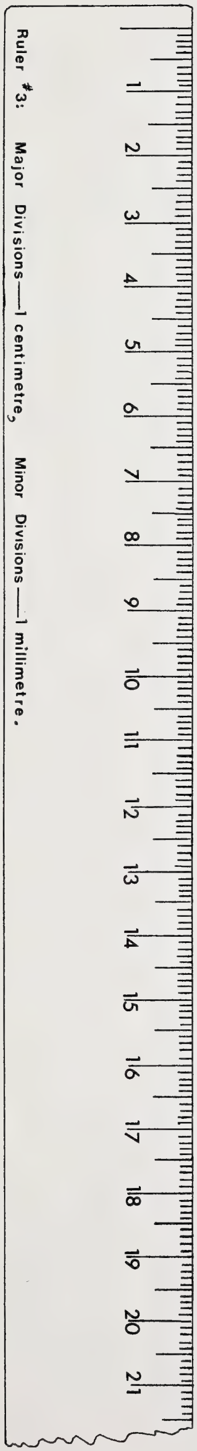
4. A building casts a shadow of 45 feet. At the same time, Bill casts a shadow which is $1\frac{1}{2}$ feet. If Bill is 6 feet tall, how tall is the building?

Scale drawings use the idea of direct variation. If the size of an object increases, its scale size also increases. First, we will consider the use of three types of rulers, before we undertake the topic of scale drawings.

The rulers on the next page are to be cut out for use in the sections on "Using a Ruler" and on "Scale Drawings".


Ruler #1: Major Divisions—1 inch, Minor Divisions—$\frac{1}{16}$ inch.


Ruler #2: Major Divisions—1 inch, Minor Divisions—$\frac{1}{10}$ inch.


Ruler #3: Major Divisions—1 centimetre, Minor Divisions—1 millimetre.

NOTE : 1 millimetre = $\frac{1}{10}$ centimetre

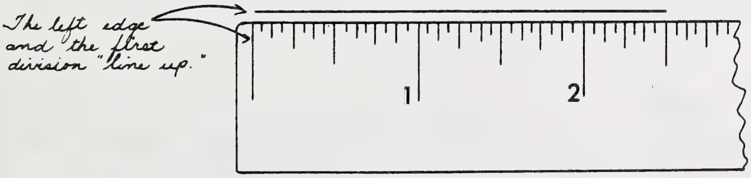
Topic Two: Using a Ruler

In order to complete the section on "Scale Drawings" we need to be able to measure lines and to draw lines of specified lengths.

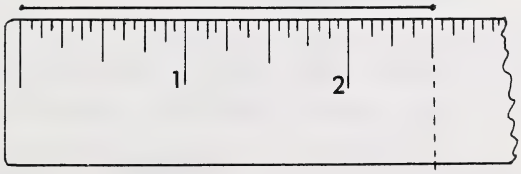
There are three types of rulers which are in common use at this time. These are (1) the ruler divided in $\frac{1}{16}$ inches, (2) the ruler divided in $\frac{1}{10}$ inches, and (3) the ruler divided in millimetres. You will find a sample of each of these types of rulers on page 6. They should be cut out and used for this lesson.

A. Measuring a Line

When measuring a line, we match the first division of the ruler with the left edge of the line.



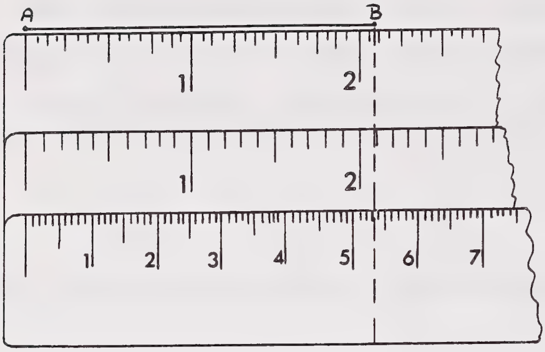
Then we see with what measurement on the ruler the right end of the line coincides.



Since the right end of the line comes closest to $2\frac{8}{16}$ inches, we say that the line is $2\frac{1}{2}$ inches long. $\left(\text{Remember } \frac{8}{16} = \frac{1}{2}\right)$. This same method is used for measuring with all types of rulers.

EXAMPLE:

Measure line AB (i) to the nearer sixteenth of an inch, (ii) to the nearer tenth of an inch, and (iii) to the nearer tenth of a centimetre.



Solution

(i) Length: $2\frac{2}{16}$ inch

(or $2\frac{1}{8}$ inch)

(ii) Length: $2\frac{1}{10}$ inch

(or 2.1 inch)

(iii) Length: 5.3 cm

(or 53 mm)

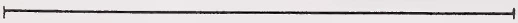
We notice that this line has roughly 2.5 times as many centimetres as inches. In fact, this should be true for any line, since there are approximately 2.5 centimetres in 1 inch.

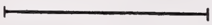
NOTE: "cm" is the symbol for centimetre.
"mm" is the symbol for millimetre.
"in." is the abbreviation for inch.

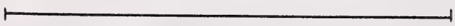
Using the three rulers, we have arrived at three different measurements. Although the measurements are different, all represent the same length.

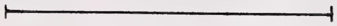
EXERCISE #2

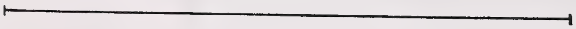
Measure each of the following lines in inches or centimetres as specified. (Give measurements in inches to the nearer sixteenth or tenth of an inch. Give measurements in centimetres to the nearer tenth of a centimetre.)

(1)  Length: ____________ cm

(2)  Length: ____________ cm

(3)  Length: ____________ cm

(4)  Length: ____________ in.

(5)  Length: ____________ cm

B. Drawing a Line of a Specific Length

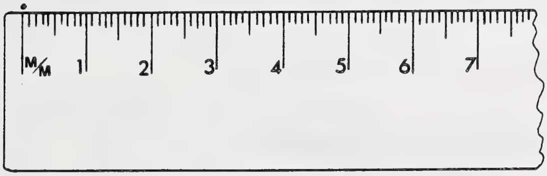
We also need to use rulers to draw lines of specified lengths.

EXAMPLE 1:

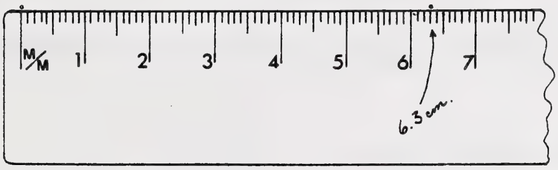
Draw a line 6.3 cm long.

Solution

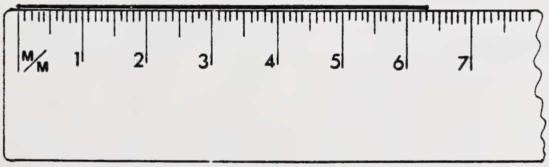
When drawing a line, first draw a dot for the left edge of the line. Then line up the first division on Ruler #3 with that dot.



Next locate the point where the line is to end (at 6.3 cm in this case) and draw a dot there.



Then using the top edge of the ruler as a guide, draw the line joining the two points.

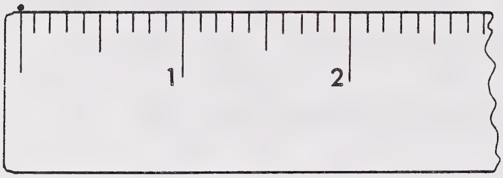


EXAMPLE 2:

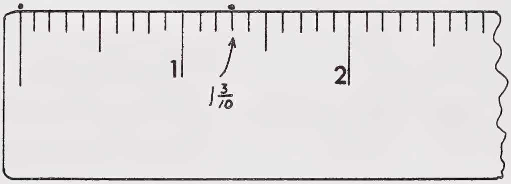
Draw a line $1\frac{3}{10}$ inches long.

Solution

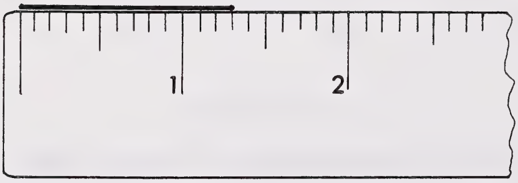
As in the last example, we draw a dot. Then we line up the dot with the first division on Ruler #2.



Now locate the other endpoint at $1\frac{3}{10}$ inches.



Finally, use the edge to draw the line joining the two points.



EXERCISE #3

Draw a line for each length given.

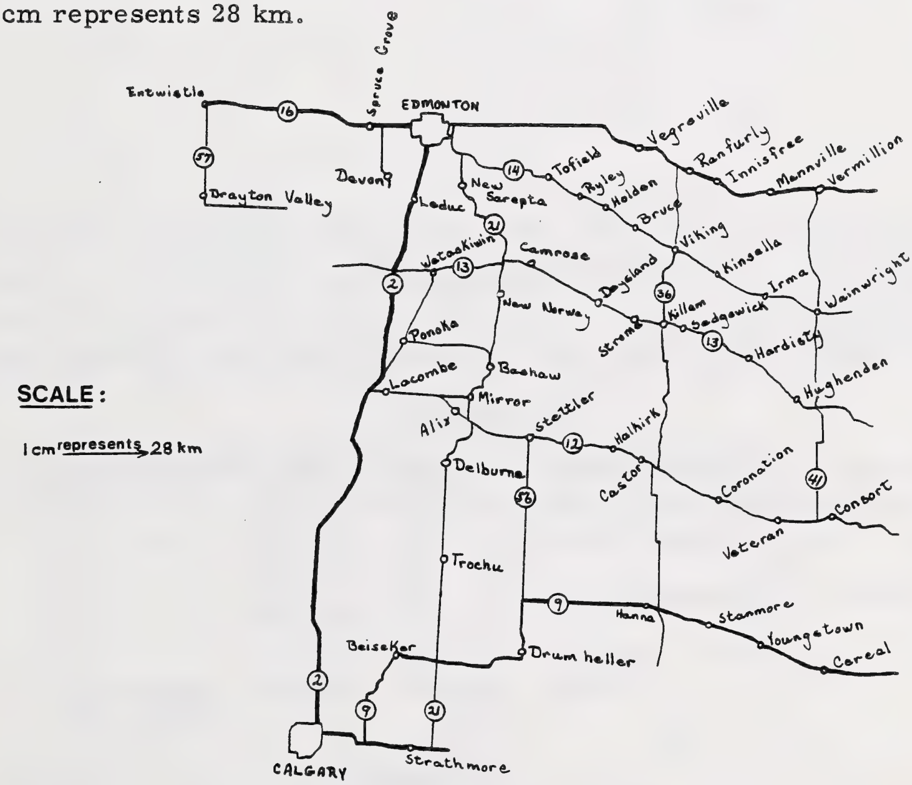
1. 3.8 cm
2. $2\frac{9}{10}$ inches
3. $4\frac{3}{16}$ inches
4. 9.2 cm

Topic Three: Scale Drawings

A. Calculating the Actual Size or Actual Distance Using a Scale Drawing

Because the actual size of an object to be drawn is often too large for the sheet of paper, we scale the drawing of the object to fit the paper. Maps and blueprints are two examples of the use of scale drawings. On a map, 1 centimetre can represent many kilometres.

Consider the map below. By measuring the length of the road between any two towns, we can calculate the approximate distance using the scale: 1 cm represents 28 km.



EXAMPLE:

To the nearer kilometre, find the approximate distance from Camrose to Strome.
(See solution on page 12.)

Solution

Using Ruler #3, we measure the distance on the map from Camrose to Strome to be 1.9 centimetres. We know that the scale of the map is 1 cm $\xrightarrow{\text{represents}}$ 28 km, so we can set up the proportion for this direct variation problem.

$$1 \text{ cm} \xrightarrow{\text{represents}} 28 \text{ km}$$

$$1.9 \text{ cm} \xrightarrow{\text{represents}} x \text{ km}$$

$$\frac{1}{1.9} = \frac{28}{x}$$

$$1 \times x = 1.9 \times 28$$

$$x = 53.2$$

$$x = 53 \text{ to the nearer km}$$

Thus, to the nearer km, the distance from Camrose to Strome is 53 km.

EXERCISE #4

1. Referring to the map on page 11, calculate the approximate distances between the given towns. Express answers to the nearer km. (Use Ruler #3 for all measurements.)

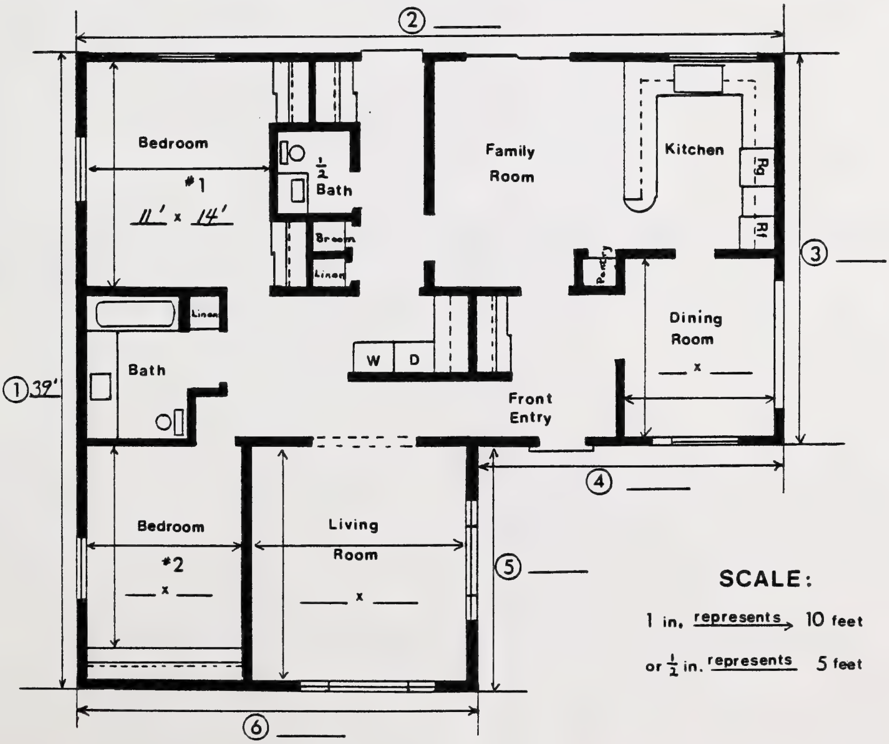
   (a) Stettler to Drumheller

   (b) Tofield to Viking

(c) Entwistle to Drayton Valley

(d) Wetaskiwin to Daysland

2. Referring to the floor plan of the house below, measure along the arrows to find the scale sizes to the nearer tenth of an inch. (Use Ruler #2.) Find the scale size of the outside walls of the house and the inside dimensions of Bedroom #1, Bedroom #2, Living Room, and Dining Room. Then calculate the actual sizes. (Use the space provided on pages 14 - 16). Mark the actual measurements in the blanks on the diagram.



Calculations

Outside Measurements

See the numbers on the diagram.

① We use Ruler #2 to find that side 1 is $3\frac{9}{10}$ inch (or 3.9 inch). Now we must find the actual size.

1 in. $\xrightarrow{\text{represents}}$ 10 ft.

3.9 in. $\xrightarrow{\text{represents}}$ y ft.

$$\frac{1}{3.9} = \frac{10}{y}$$

$$1 \times y = 3.9 \times 10$$

$$y = 39 \text{ ft.}$$

This is the symbol for "feet".

We enter 39' in blank ① on the diagram.

②

③

④

⑤

⑥

Inside Measurements

Bedroom #1

Measure of one direction: 1.4 inch

$$1\text{ in.} \xrightarrow{\text{represents}} 10\text{ ft.}$$

$$1.4\text{ in.} \xrightarrow{\text{represent}} x\text{ ft.}$$

$$\frac{1}{1.4} = \frac{10}{x}$$

$$1 \times x = 1.4 \times 10$$

$$x = 14\text{ feet}$$

Measure of the other direction: 1.1 inch

$$1\text{ in.} \xrightarrow{\text{represents}} 10\text{ ft.}$$

$$1.1\text{ in.} \xrightarrow{\text{represent}} n\text{ ft.}$$

$$\frac{1}{1.1} = \frac{10}{n}$$

$$1 \times n = 1.1 \times 10$$

$$n = 11\text{ feet}$$

We enter 11' × 14' in the blanks on the diagram in Bedroom #1. (When writing the dimensions of a room, the smaller measurement is usually written first.)

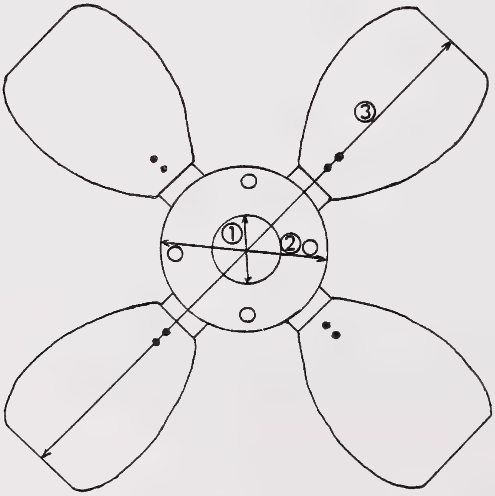
Bedroom #2

Living Room

Dining Room

In each of the above calculations, the scale measurement was multiplied by what number? ________

3. In making its basic plans for an automobile, a European firm made scale drawings of all the parts to be manufactured. Below see the scale drawing of a fan. Using the indicated scale, calculate the actual dimensions (in centimetres) of the lengths marked on the fan. (Use Ruler #3).



SCALE

10 mm $\xrightarrow{\text{represents}}$ 8 cm

Calculations

See the numbers on the diagram on page 16.

①

②

③

B. Making a Scale Drawing

This process is the reverse of the one taken in the previous section. Here we are given the actual measurements and the scale. Using this information, we calculate the dimensions for the drawing. Then we can draw the "scaled" version of the diagram.

EXAMPLE:

Mr. Jordan's rectangular back yard measures 50 feet wide by 20 feet deep. Along the west side, there is a strip flower bed which is 2 feet wide. In the centre of the yard, Mr. Jordan has placed a circular swimming pool 8 ft. in diameter. At the south-east corner of the yard, Mrs. Jordan has her vegetable garden which is 25 feet wide and 4 feet deep. Using Ruler #2 and the scale $\frac{1}{10}$ inch $\xrightarrow{\text{represents}}$ 1 foot, draw the Jordan's back yard.

Solution

First we make all calculations.

(a) Width of the Lot

$\frac{1}{10}$ in. $\xrightarrow{\text{represents}}$ 1 ft.
w in. $\xrightarrow{\text{represents}}$ 50 ft.

$$\frac{\frac{1}{10}}{w} = \frac{1}{50}$$

$$w \times 1 = \frac{1}{10} \times 50$$

$$w = 5 \text{ inches}$$

Thus, the width of the lot should be drawn 5 inches long on the diagram.

(b) Depth of the Lot

$\frac{1}{10}$ in. $\xrightarrow{\text{represents}}$ 1 ft.
d in. $\xrightarrow{\text{represents}}$ 20 ft.

$$\frac{\frac{1}{10}}{d} = \frac{1}{20}$$

$$d \times 1 = \frac{1}{10} \times 20$$

$$d = 2 \text{ inches}$$

Thus the depth of the lot should be drawn 2 inches long on the diagram.

(c) Width of the Flower Bed

$\frac{1}{10}$ in. $\xrightarrow{\text{represents}}$ 1 ft.
f in. $\xrightarrow{\text{represents}}$ 2 ft.

$$\frac{\frac{1}{10}}{f} = \frac{1}{2}$$

← COMPLETE

Thus, the width of the flower bed should be drawn ______ in. wide on the diagram.

(d) Diameter of the Pool

$\frac{1}{10}$ in. $\xrightarrow{\text{represents}}$ 1 ft.
p in. $\xrightarrow{\text{represents}}$ 8 ft.

$$\frac{\frac{1}{10}}{p} = \frac{1}{8}$$

$$p \times 1 = \frac{1}{10} \times 8$$

$$p = \frac{8}{10} \text{ inch}$$

Thus the pool should be drawn with a diameter of $\frac{8}{10}$ inch.

(e) Width of the Garden

$\frac{1}{10}$ in. $\xrightarrow{\text{represents}}$ 1 ft.
g in. $\xrightarrow{\text{represents}}$ 25 ft.

$$\frac{\frac{1}{10}}{g} = \frac{1}{25}$$

(Cont'd on next page.)

(f) Depth of the Garden

$\frac{1}{10}$ in. $\xrightarrow{\text{represents}}$ 1 ft.
x in. $\xrightarrow{\text{represents}}$ 4 ft.

$$\frac{\frac{1}{10}}{x} = \frac{1}{4}$$

(Cont'd on next page.)

← COMPLETE

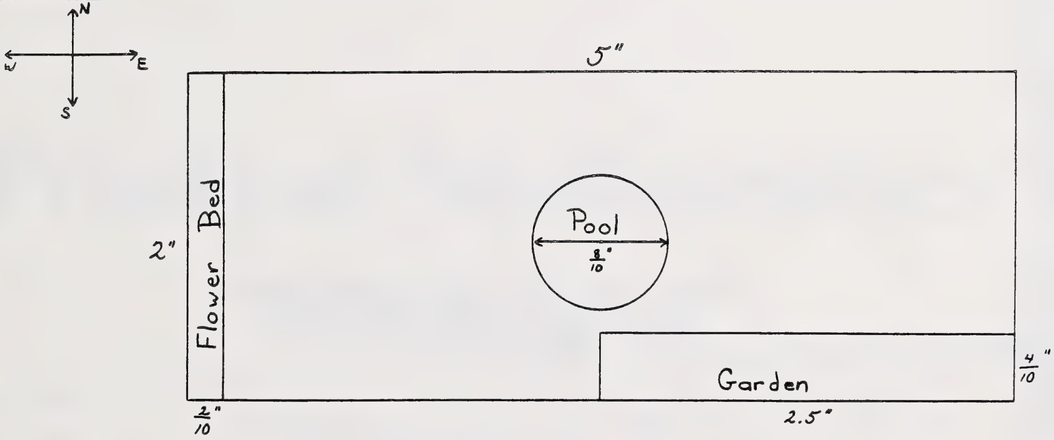
Thus, the width of the garden should be drawn as __________ inches on the diagram.

$$x \times 1 = \frac{1}{10} \times 4$$

$$x = \frac{4}{10} \text{ inch}$$

Thus the garden should be drawn $\frac{4}{10}$ inch deep on the diagram.

Diagram



EXERCISE #5

Make a scale drawing of a football field. The following must be incorporated in the drawing:

Length of field (excluding the end zones):	110 yards
Width of field:	65 yards
Length of each end zone:	25 yards

Yard lines: Spaced every 5 yards along the field

Scale: 10 yards $\xrightarrow{\text{is represented by}}$ 1 cm

Calculations

Diagram

Solutions to Self-Correcting Exercises

Self-Correcting Exercise #1, page 3

Both units on the left are "dozen."

1. $1 \text{ dozen} \xrightarrow{\text{costs}} 85¢$

$\frac{1}{2} \text{ dozen} \xrightarrow{\text{costs}} n¢$

$$\frac{1}{\frac{1}{2}} = \frac{85}{n}$$

or

Both units on the left are "donuts."

$12 \text{ donuts} \xrightarrow{\text{cost}} 85¢$

$6 \text{ donuts} \xrightarrow{\text{cost}} n¢$

$$\frac{12}{6} = \frac{85}{n}$$

2. $1000 \text{ bearings} \xrightarrow{\text{weigh}} 300 \text{ kg}$

$3425 \text{ bearings} \xrightarrow{\text{weigh}} n \text{ kg}$

$$\frac{1000}{3425} = \frac{300}{n}$$

3. $320 \text{ km} \xrightarrow{\text{is covered in}} 4 \text{ hours}$

$n \text{ km} \xrightarrow{\text{is covered in}} 7 \text{ hours}$

$$\frac{320}{n} = \frac{4}{7}$$

End of Lesson 4

Practical Mathematics
Module 4
Lesson 5

MORE ON VARIATION

In the last lesson, we studied direct variation. Now let's look at inverse variation.

Topic One: Inverse Variation

> When two quantities VARY INVERSELY, an increase in one quantity causes a certain decrease in the other quantity. INVERSE VARIATION also occurs when a decrease in one quantity creates a certain increase in the other quantity.

EXAMPLES:

1. The more pressure on a fixed amount of gas, the less space it occupies. (The gas has been squeezed into a smaller space.)

   e.g., If the gas occupies 50 cm^3 at 20 mm of pressure, it will occupy 25 cm^3 at 40 mm of pressure.

   "cm^3 is the symbol for "cubic centimetre."
   "mm" is the symbol for "millimetre."

   Thus, as the pressure increases, the amount of space decreases.

2. The fewer men who work on a job, the longer it takes to complete the job.

   e.g., If it takes 4 men 4 hours to complete a job, it would take 2 men 8 hours to complete the same job.

   Thus, as the number of men working decreases, the time required to complete the job increases.

When given an inverse variation problem with a missing element, we want to use a proportion to solve for the unknown quantity. When solving a direct variation problem, the use of the proportion is straight forward. This is not true in the case of inverse variation. To see this, study the following example of inverse variation.

EXAMPLE:

The speed that a car travels varies inversely as the time it takes to go a fixed distance. (This means that the faster the car goes, the less time it takes to go a fixed distance.)

e.g. To go a fixed distance of 100 km
- it takes 5 hours for a car travelling 20 km/h
- it takes $2\frac{1}{2}$ hours for a car travelling 40 km/h
- it takes 2 hours for a car travelling 50 km/h

NOTE: In every case the product of the rate and the time is constant, since the product always equals 100 km.

For example,

$$5 \times 20 = 2\frac{1}{2} \times 40.$$

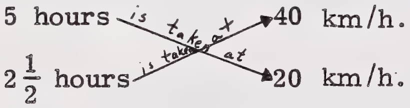
each side of the equation is the product of the rate and the time

Therefore, the proportion must be set up

$$\frac{5}{2\frac{1}{2}} = \frac{20}{40}$$

to give the above equal cross products.

From the proportion, we arrive at the initial setup:

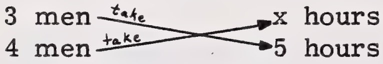


We see that the related quantities are not on the same line, as in the case of direct variation. The arrows joining the related quantities cross each other.

EXAMPLES:

1. If it takes 3 men 5 hours to build a brick wall, how long will it take 4 men to do the job?

Solution (Inverse Variation)



Because this is inverse variation, the related quantities are not on the same line.

$$\frac{3}{4} = \frac{x}{5}$$

← COMPLETE

It will take the 4 men ______ hours to build the brick wall.

(NOTE: It takes less time for more men to do the job.)

2. The current in a wire varies inversely as the resistance of the conductor. If a conductor with a resistance of 5.6 ohms has a current of 25 amperes, how much current is in a wire conductor with a resistance of 3.4 ohms?

Solution (Inverse Variation)

$$\begin{tikzcd} 25 \text{ amperes} \arrow[dr, "for"] & 3.4 \text{ ohms} \\ z \text{ amperes} \arrow[ur, "for"'] & 5.6 \text{ ohms} \end{tikzcd}$$

$$\frac{25}{z} = \frac{3.4}{5.6}$$

$$z \times 3.4 = 25 \times 5.6$$

$$\frac{z \times \overset{1}{\cancel{3.4}}}{\underset{1}{\cancel{3.4}}} = \frac{25 \times \overset{2.8}{\cancel{5.6}}}{\underset{1.7}{\cancel{3.4}}}$$

$$z = \frac{70}{1.7}$$

$$z = 41.2 \text{ amperes}$$

The wire with a resistance of 3.4 ohms has a current of 41.2 amps. (NOTE: For a smaller resistance, the current is larger.)

In our examples on inverse variation, we find that for each pair of related quantities in the proportion, one quantity occupies the numerator of one ratio, and the other occupies the denominator of the second ratio.

Self-Correcting Exercise #1

For each of the following inverse variation problems, set up the proportion needed to solve it. Use n as the variable for each problem. (You do not need to solve the proportion.)

1. On their vacation, the Bells travelled $4\frac{1}{2}$ hours at an average speed of 80 km/h. How long would it have taken them to cover the same distance if their average speed was increased to 90 km/h?

2. 3 men can lay a tile floor in 5 hours. How long would it have taken 2 men to complete the same job?

3. In an electronic circuit, an 8.9 ohm resistor has a current of 21 amperes. If the original resistor is replaced by a 10.5 ohm resistor, how much current passes through the 10.5 ohm resistor?

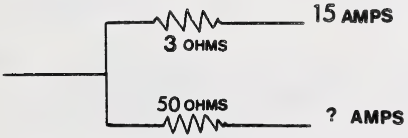
Check your answers to this exercise against the solutions on page 17.

EXERCISE #1

For each of the following inverse variation problems, set up the proportion, then solve for the unknown quantity. Complete each problem with an answer sentence.

1. On Friday night, John made the trip from Edmonton to Calgary in $3\frac{1}{2}$ hours. His average speed was 80 km / h. During a Sunday snowstorm, the road condition became poor. If the trip back to Edmonton Sunday evening took $5\frac{1}{2}$ hours, what was his average speed? (Round your answer to the nearer tenth.)

2. If it takes 6 men 12 hours to paint a house, how long will it take 10 men to complete the job?



3. In an electronic circuit, two resistors are connected in parallel. The current which passes through a resistor varies inversely as the resistance. One resistor is 3 ohms and has a current of 15 amperes. How much current is in the 50-ohm resistor?

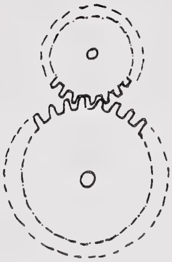
Inverse variation is encountered in many real-life situations other than those just described. Some problems involving gear ratios, drive belts, and levers are solved using the theory of inverse variation.

Topic Two: More Applied Problems Involving Inverse Variation

A. Gear Ratios

In considering two gears that are meshed, we know that a gear with more teeth makes fewer revolutions per minute (r.p.m.) than a gear with fewer teeth; thus, the number of teeth varies inversely as the number of revolutions.

EXAMPLE:



A gear having 72 teeth makes 750 r.p.m. It meshes with another gear which makes 1000 r.p.m. How many teeth does the second gear have?

Solution (Inverse)

72 teeth make make 1000 r.p.m.
y teeth make 750 r.p.m.

$$\frac{72}{y} = \frac{1000}{750}$$

← COMPLETE

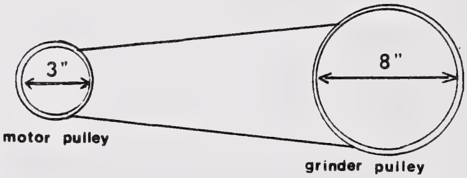
The second gear has ______ teeth.

← This number should be less than 72.

B. Drive Belts (Pulleys)

When considering drive belts, the larger the pulley, the fewer r.p.m.'s it makes. Thus, the speed of a pulley varies inversely as its diameter.

EXAMPLE:



A grinder pulley with an 8-inch diameter is driven off a motor pulley with a 3-inch diameter. If the motor pulley turns at a rate of 1500 r.p.m., at what rate does the grinder pulley rotate?

Solution (Inverse)

3 inches rotates at z r.p.m.
8 inches rotates at 1500 r.p.m.

$$\frac{3}{8} = \frac{z}{1500}$$

← COMPLETE

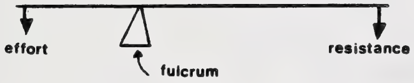
So the grinder pulley rotates at ________ r.p.m.

← this number should be less than 1500

C. Levers

There are 3 classes of levers. For all levers, the forces which are applied vary inversely as their distances from the fulcrum (or balance point).

Class I Lever



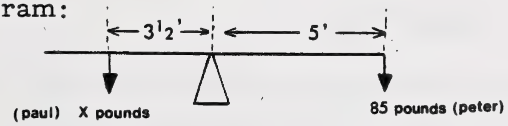
The balance point or fulcrum is between the effort and the resistance. (The positions of the effort and resistance can be interchanged.)

EXAMPLE:

Peter and Paul can balance a teeter totter when Paul is $3\frac{1}{2}$ feet from the fulcrum and Peter is 5 feet from the fulcrum. If Peter weighs 85 pounds, how much does Paul weigh?

Solution (Inverse)

Diagram:



$$\begin{tikzcd} x \text{ pounds} \arrow[dr, "at"] & 5 \text{ feet} \\ 85 \text{ pounds} \arrow[ur, "at"'] & 3\frac{1}{2} \text{ feet} \end{tikzcd}$$

$$\frac{x}{85} = \frac{5}{3\frac{1}{2}}$$

$$x \times 3\frac{1}{2} = 85 \times 5$$

$$\frac{x \times \overset{1}{\cancel{\frac{7}{2}}}}{\underset{1}{\cancel{\frac{7}{2}}}} = \frac{85 \times 5}{\frac{7}{2}}$$

$$x = 425 \div \frac{7}{2}$$

$$x = 425 \times \frac{2}{7}$$

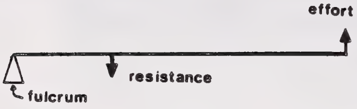
Here we are multiplying by the reciprocal of the divisor.

$$x = \frac{850}{7}$$

$$x = 121\frac{3}{7} \text{ pounds}$$

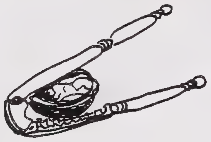
Paul weighs 121 pounds (to the nearer pound).

Class II Lever



The fulcrum is on the end of the lever and the resistance is between the fulcrum and the effort. (The positions of the fulcrum and the effort can be interchanged.)

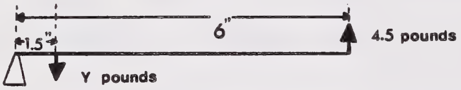
EXAMPLE:



How much force was needed to crack a walnut if Steve applied a 4.5 pound force at the end of the handles? The nut was placed 1.5 inches from the hinge. The handles are 6 inches long.

Solution (Inverse)

Diagram:



(all distances are measured from the fulcrum

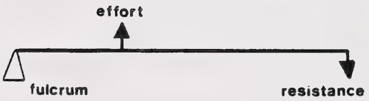
4.5 pounds at $1\frac{1}{2}$ inches
y pounds at 6 inches

$$\frac{4.5}{y} = \frac{1.5}{6}$$

← COMPLETE

_____________ pounds of force were needed to crack the walnut.

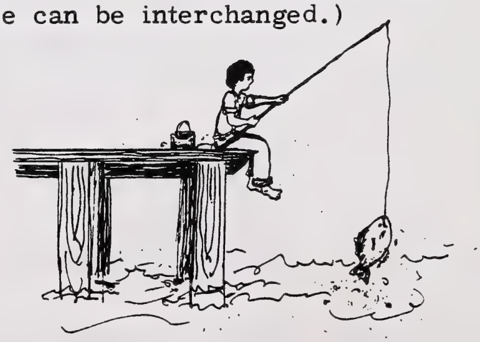
Class III Lever



The fulcrum is on the end of the lever and the effort is between the resistance and the fulcrum. (The positions of the fulcrum and the resistance can be interchanged.)

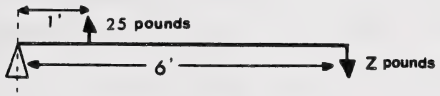
EXAMPLE:

While fishing off a pier, Frank had his hands 1 foot apart on his 6 foot pole. What is the heaviest fish he can lift from the water, if the applies an upward force of 25 pounds while keeping the hand at the end of the pole stationary.

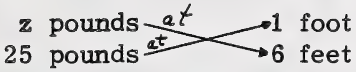


Solution (Inverse)

Diagram:



"ALL DISTANCES ARE MEASURED FROM THE FULCRUM."



$$\frac{z}{25} = \frac{1}{6}$$

← COMPLETE

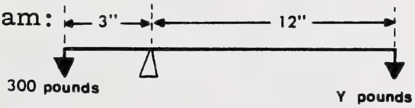
With the 25 pounds of force, Frank can lift a ______ pound fish from the water.

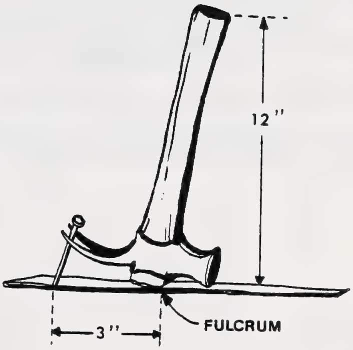
EXERCISE #2

Solve the following problems involving inverse variation.

1. If a nail has a holding force of 300 pounds, how much force must be exerted on the end of a 12" hammer to pull out the nail? (The nail is 3" from the point where the hammer head rests on the board.)

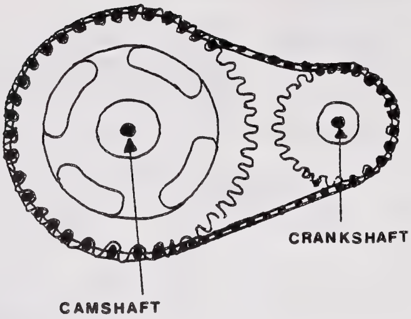
Diagram:





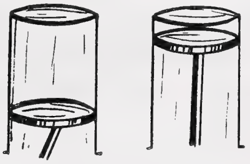
(this is a class 1 lever)

2. A camshaft is driven by a timing chain. The camshaft sprocket has 36 teeth and the crankshaft sprocket has 18 teeth. When cruising at 65 m.p.h., the crankshaft turns at a rate of 2,500 r.p.m. At what rate does the camshaft turn?

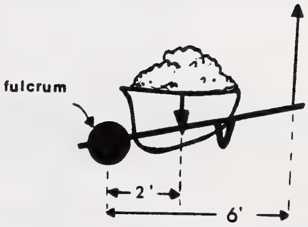


3. How much upward force must Jack apply with his left hand to lift a 30 pound shovel of snow? His hands are 2 feet apart and his right hand is acting as the fulcrum. There is $2\frac{1}{2}$ feet between his left hand and the center of the load.

4. In a cylinder of a car's engine, the pressure varies inversely as the volume of the cylinder. The volume is 4.8 cubic inches at the bottom of the stroke and 0.6 cubic inches at the top of the stroke. If the pressure is 14 pounds per square inch at the bottom of the stroke, what is the pressure at the top of the stroke?



5. In a wheelbarrow, we have a 250 pound load centered 2 feet from the front axle. If Dan picks up the wheelbarrow by the handes 6 feet from the front axle, how many pounds of force must he apply? (Express the answer as a mixed number.)



(THIS IS A CLASS II LEVER)

Diagram:

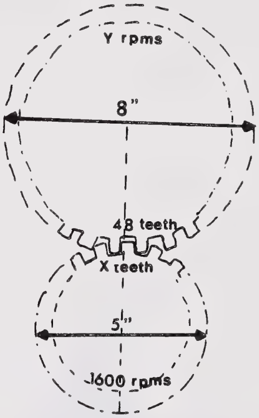
Topic Three: Problems Involving Direct or Inverse Variation

We have been considering direct and inverse variation problems. Up to this point the problems have been neatly divided so that one section considers only exercises involving direct variation, while another section considers only inverse variation.

Unfortunately, the world is not so neatly separated. When you encounter a variation problem, you must examine the situation to determine whether the variation is direct or inverse. Study the following examples.

EXAMPLE 1:

An 8" gear has 48 teeth and it is meshed with a 5" gear. How many teeth does the smaller gear have?



Type of Variation

To be meshed, the size of the teeth on both gears must be the same. Thus, the larger the gear, the more teeth it will have. Therefore, the variation is direct.

Solution

$$8 \text{ inches} \xrightarrow{\text{has}} 48 \text{ teeth}$$
$$5 \text{ inches} \xrightarrow{\text{has}} x \text{ teeth}$$

Remember - In direct variation, the arrows joining the related quantities are parallel.

$$\frac{8}{5} = \frac{48}{x}$$

$$8 \times x = 5 \times 48$$

$$\frac{\overset{1}{\cancel{8}} \times x}{\underset{1}{\cancel{8}}} = \frac{5 \times \overset{6}{\cancel{48}}}{\underset{1}{\cancel{8}}}$$

$$x = 30 \text{ teeth}$$

The 5" gear has 30 teeth.

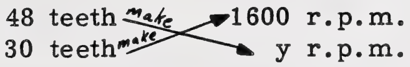
EXAMPLE 2: (See the same illustration used for EXAMPLE 1)

A gear with 48 teeth is meshed with a gear having __________ teeth (see solution to EXAMPLE 1). If the 5" gear is rotating at 1600 r.p.m., how fast is the 8" gear turning?

Type of Variation

In the time the smaller gear rotates once, the larger gear has not made a full rotation. In fact, the more teeth the larger gear has, the less often it rotates. Thus, the variation is inverse.

Solution



(Remember – In inverse variation the arrows joining the related quantities cross.)

$$\frac{48}{30} = \frac{1600}{y}$$

COMPLETE

Thus the 8" gear makes __________ r.p.m.

EXERCISE #3

For each of the following, tell whether the variation is direct or inverse. Then solve the problem. Complete each problem with a statement.

1. If it takes 2 men 3 hours to paint 756 square feet of wall area, how long will it take these men to paint 2100 square feet of wall area? (Express your answer to the nearer tenth.)

   Type of Variation

   As the number of square feet of wall area increase, does the time required to paint the wall increase or decrease? ____________________

   Is the variation direct or inverse? ____________________

   Solution

2. It took Bill 16 hours to build a fieldstone walkway. How long would the job have taken if he had gotten two friends to help? (Express answer as a mixed number.)

Type of Variation

As the number of workers increase, does the time needed to build the walk increase or decrease? ____________________
Is the variation direct or inverse? ______________________
Solution

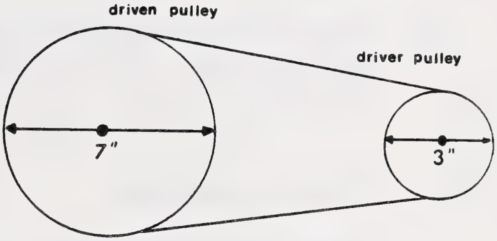
3. Steve's car averaged 6.3 km per litre on a trip to Slave Lake. The distance from Edmonton to Slave Lake is 248 km. How much gas did Steve use on his trip? (Round answer to the nearer tenth of a litre.)

Type of Variation

Is more or less gas needed the farther Steve drives? ____________________

Is the variation direct or inverse? ____________________

Solution



4. We have a belted pulley system in which the driver pulley rotates at 1800 r.p.m. How fast is the driven pulley turning? (The driver pulley has a diameter of 3" and the driven pulley has a diameter of 7".) (Round your answer to the nearer 100 r.p.m.)

Type of Variation

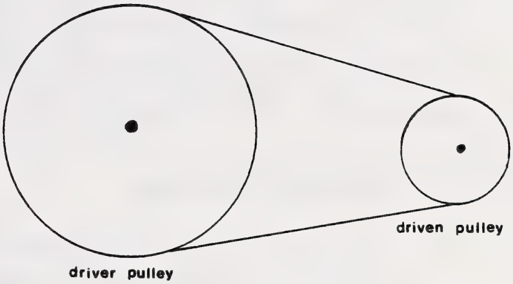
Is the larger pulley turning faster or slower than the smaller pulley?

____

Is the variation direct or inverse? ____________________

Solution

5. In another belted pulley system, when the driver pulley rotates at 1200 r.p.m., the driven pulley rotates at 3200 r.p.m. If the speed of the driver pulley is changed to 1000 r.p.m., what is the new rate of the driven pulley? (Round answer to the nearer 100 r.p.m.)



Type of Variation

As the speed of the driver pulley decreases, does the speed of the driven pulley increase or decrease? ____________________

Is the variation direct or inverse? ____________________

Solution

6. Hank and Jim left Swift Current for North Battleford in separate automobile
Jim made the trip in 5 hours at an average speed of 64 km/h. If Hank averaged 90 km/h, how long did he spend on the road? (Express the answer as a mixed number in lowest terms.)

Type of Variation

If the speed increases, does the time required to make a trip increase or decrease? ____________________

Is the variation direct or inverse? ________________________

Solution

7. Joe and Sarah were on their way from Edmonton to Vancouver for holidays. They had travelled the 808 kilometres to Kamloops in 10 hours. If they continue at the same average speed, (a) how long will it take them to travel the 432 kilometres from Kamloops to Vancouver? and (b) what is the total time Joe and Sarah spent on the road? (Answer should be expressed to one decimal place.)

(a) Type of Variation

As the number of kilometres travelled increases, has more or less time been spent on the road? ________________________

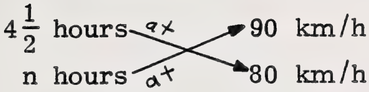
Is the variation direct or inverse? ____________________

Solution

(b) Solution

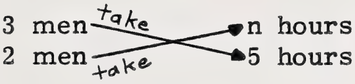
Solutions to Self-Correcting Exercise #1, page 3

1.



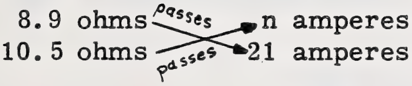
$$\frac{4\frac{1}{2}}{n} = \frac{90}{80}$$

2.



$$\frac{3}{2} = \frac{n}{5}$$

3.



$$\frac{8.9}{10.5} = \frac{n}{21}$$

End of Lesson 5

Practical Mathematics
Module 4
Lesson 6

POWERS AND SQUARE ROOTS

Topic One: Powers

A. Defining a Power

Consider these products: 2×2, $3 \times 3 \times 3 \times 3$, $g \times g \times g \times g \times g$. It can be tiring to write out all the factors of a product. To solve this situation a shorthand method of writing products with equal factors was devised many years ago.

$2 \times 2 = 2^2$	$3 \times 3 \times 3 \times 3 = 3^4$	$g \times g \times g \times g \times g = g^5$
("Two squared")	("Three to the fourth")	("g to the fifth")

The shorthand form is called a POWER. So 2^2, 3^4, and g^5 are called powers. Each power has two parts: (1) the BASE is the number used as a factor, and (2) the EXPONENT indicates how many factors in the product. (The exponent is a small number written slightly above and to the right of the base.)

EXAMPLES:

exponent
5^3
("five cubed")
base

This power tells us that 5 is used as a factor 3 times, or $5^3 = 5 \times 5 \times 5$.

exponent
a^{10}
("a to the tenth")
base

This power tells us that a is used as a factor 10 times, or $a^{10} = a \times a \times a \times a \times a \times a \times a \times a \times a \times a$.

Self-Correcting Exercise #1

1. Write each of the following products as a power.

(a) $4 \times 4 \times 4 =$ ______ (b) $a \times a \times a \times a \times a =$ ______ (c) $7 \times 7 =$ ______

(d) $12 \times 12 \times 12 \times 12 =$ ______ (e) $f \times f \times f \times f \times f \times f =$ ______

(f) $r \times r \times r \times r \times r \times r \times r \times r \times r =$ ______

2. Write each of the following powers as a product of factors.

(a) $2^7 =$ ______________ (b) $6^2 =$ ______________

(c) $q^3 =$ ______________ (d) $5^4 =$ ______________

(e) $1^5 =$ ______________ (f) $t^{11} =$ ______________

Answers to this exercise should be checked with the solutions on page 23.

B. Evaluating Powers

To evaluate a power with a numerical base (i.e., the base is a number):

1. Express the power as a product of factors.
2. Find the product by multiplying groups of two factors until the product is reached.

EXAMPLES:

We arrange the factors in groups of two to find the product.

$5^4 = (5 \times 5) \times (5 \times 5) = (25 \times 25) = 625$

$2^7 = (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times 2 = (4 \times 4) \times (4 \times 2) = (16 \times 8) = 128$

$6^2 = (6 \times 6) = 36$

$3^3 = (3 \times 3) \times 3 = (9 \times 3) = 27$

Self-Correcting Exercise #2

Evaluate the following powers.

1. $0^6 =$ ____________________
2. $2^5 =$ ____________________
3. $7^2 =$ ____________________
4. $4^3 =$ ____________________
5. $6^4 =$ ____________________

Answers to this exercise should be checked with the solutions on page 23.

EXERCISE #1

1. Write each of the following powers as a product of factors, then evaluate.

(a) 3^2 = $(3 \times 3) = 9$ (b) 1^3 = ______

(c) 2^4 = ______ (d) 4^2 = ______

(e) 5^3 = ______ (f) 3^4 = ______

2. Write each of the following products as a power.

(a) $a \times a \times a$ = a^3 (b) $r \times r$ = ______

(c) $s \times s \times s$ = ______ (d) $19 \times 19 \times 19 \times 19 \times 19$ = ______

(e) $21 \times 21 \times 21$ = ______ (f) $4 \times 4 \times 4 \times 4 \times 4 \times 4$ = ______

C. Perfect Squares

A PERFECT SQUARE is a number which can be written as a power with an exponent of 2.

4 is a perfect square, because it can be written as the power 2^2.

16 is a perfect square, because it can be written as the power 4^2.

15 is not a perfect square, because it cannot be written as a power with an exponent of 2.

Complete the table below.

Table of Squares of the Numbers 0 - 25

NUMBER	SQUARE OF NUMBER	FACTORS	PRODUCT (PERFECT SQUARE)
0	0^2	0×0	0
1	1^2	1×1	1
2	2^2	2×2	4
3	3^2	3×3	9

Continued on next page.

NUMBER	SQUARE OR NUMBER	FACTORS	PRODUCT (PERFECT SQUARE)
4			
5			
6			
7			
8			
9			
10			
11	11^2	11 × 11	121
12			
13			
14			
15			
16			
17	17^2	17 × 17	289
18			
19			
20			
21			
22			
23	23^2	23 × 23	529
24			
25			

Self-Correcting Exercise #3

For each of the following, identify the expression as being a "perfect square" or "not a square".

1. 115 not a square   2. 121 perfect square

3. 5^3 ________   4. a^2 ________

5. 3^2 ________   6. 36 ________

7. 225 ________   8. 23^2 ________

9. 1903^5 ________   10. 194^2 ________

11. 2^7 ________   12 169 ________

Evaluate only those powers which are perfect squares. Write "not a square" for those powers which are not perfect squares.

13. $3^5 =$ ________   14. $16^2 =$ ________

15. $22^2 =$ ________   16. $19^2 =$ ________

17. $2^{15} =$ ________   18. $59^2 =$ ________

19. $1398^{23} =$ ________   20. $40^2 =$ ________

21. $47^7 =$ ________   22. $60^2 =$ ________

Check your answers against the solutions on page 24 of this lesson.

Now let's consider some sets of perfect squares.

1. $(2)^2 = 4$ $(20)^2 = 400$ $(200)^2 = 40,000$

2. $(5)^2 =$ ____ $(50)^2 =$ ____ $(500)^2 =$ ____

3. $(14)^2 = 196$ $(140)^2 =$ ____ $(1400)^2 =$ ____

We notice that for each zero in the base of the power, there are two zeros in the perfect square.

Self-Correcting Exercise #4

In each case, tell how many zeros are in the perfect square.

Number of zeros   Number of zeros

1. $(139600)^2$ 4 zeros   2. $(590)^2$ ______

3. $(7000)^2$ ______   4. $(1420)^2$ ______

5. $(8410)^2$ ______   6. $(190000)^2$ ______

Fill in the blanks below. (You may refer to the table of perfect squares on pages 3 - 4.)

7. $64 = (\underline{8})^2$, $6400 = (\_\_\_\_\_)^2$ $640000 = (\_\_\_\_\_\_\_)^2$

8. $9 = (\_\_)^2$, $90000 = (\_\_\_\_\_)^2$ $9000000 = (\_\_\_\_\_\_\_)^2$

9. $225 = (\_\_)^2$, $22500 = (\_\_\_\_\_)^2$ $2250000 = (\_\_\_\_\_\_\_)^2$

10. $529 = (\_\_)^2$, $52900 = (\_\_\_\_\_)^2$ $529000000 = (\_\_\_\_\_\_\_)^2$

Answers to this exercise should be checked with the solutions on page 24.

Topic Two: Scientific Notation

At times we may be faced with writing very large numbers — e.g. the earth is 93,000,000 miles from the sun. Scientists have developed a shorthand way to write very large numbers. This method is called scientific notation.

EXAMPLES:

189,000,000,000 can be written as 1.89×10^{11}

350,000,000,000,000 can be written as 3.5×10^{14}.

> SCIENTIFIC NOTATION is a number written as two factors. The first factor is greater than or equal to one and it is less than ten. The second factor is a power with a base of 10.

When we examine the above examples, we see that the definition of scientific notation has been satisfied. We now need to know the rules for writing a number in scientific notation, and how to change a number from scientific notation back into its decimal form.

To write a number in scientific notation, we:

1. Move the decimal point to the left to obtain a number which is between 1 and 10, or it is 1. This is the first factor.
2. Multiply the first factor by the power whose base is 10 and whose exponent corresponds to the number of places which the decimal has been moved to obtain the first factor.

EXAMPLES:

Write the following decimal numbers in scientific notation. (Remember if there is no decimal point in the number, it is understood to be immediately to the right of the last digit.)

93,000,000 miles = 9.3×10^7 miles

Since we move the decimal point 7 places to the left to obtain the first factor, the exponent of the power is 7.

5,243,000,000,000 = 5.243×10^{__}

Since the decimal point moves __ places to the left, the exponent is __.

To change a number from scientific notation into its decimal form, we:

1. Move the decimal as many places to the right as indicated by the exponent of the power with base 10. (Add zeros as needed for placeholders.)
2. Drop the second factor.

EXAMPLES:

5.234×10^6 represents the number 5234000.

Three zeros are needed as placeholders so the decimal can move 6 places to the right.

7.92×10 represents the number 79.2

When no exponent is written it is understood to be 1.

The decimal moves one place to the right.

8.753×10^{12} represents the number 8,753,000,000,000.

Nine zero placeholders are needed to enable the decimal to move __ places to the right.

Self-Correcting Exercise #5

1. Check each of the following expressions that is a number written in scientific notation. (Remember that the first factor of a number in scientific notation is greater than or equal to one and is less than 10; and the second factor is a power with 10 as its base.)

(a) 3.4×10^5 ____________
(b) 13.5×10^2 ____________
(c) 2.0×10^8 ____________
(d) 7.1×10 ____________
(e) 0.1×10^3 ____________
(f) 0.79×10^4 ____________
(g) 6.23×10^4 ____________
(h) 0.439×10^2 ____________
(i) 3.6×5^{10} ____________
(j) 10×10^3 ____________

2. Write each of the following decimal numbers in scientific notation.

(a) 156,000 = ____________
(b) 97.3 = ____________
(c) 728 = ____________
(d) 2,500 = ____________
(e) 1,687,000 = ____________
(f) 7,235,000,000 = ____________
(g) 1.3 = ____________
(h) 1,090,000 = ____________
(i) 465.2 = ____________
(j) 24 = ____________

3. Rewrite each of the following as decimal numbers.

(a) 7.77×10^5 = ____________
(b) 34.8×10^{13} = ____________
(c) 1.2×10^8 = ____________
(d) 6.1×10^4 = ____________
(e) 1.4325×10^3 = ____________
(f) 2.8853×10^6 = ____________
(g) 8.7056×10^9 = ____________
(h) 5×10^2 = ____________
(i) 4.05×10^2 = ____________
(j) 8.2×10 = ____________

Answers to this exercise should be checked against the solutions on page 24.

EXERCISE #2

Evaluate the following powers.

1. 5^3 (5×5)×5 = 125 2. 19^2 = ____________ 3. 31^2 = ____________

4. 25^2 = ____________ 5. $(50)^2$ = ____________ 6. 6^3 = ____________

Write each of the following in scientific notation.

7. 1200 = ________________ 8. 173,000 = ________________

9. 5,263,000,000 = ________________ 10. 9,834 = ________________

11. 65,900,000 = ________________ 12. 726 = ________________

Rewrite each of the following as a decimal number.

13. 1.83×10^{11} = ____________ 14. 5.4×10^{6} = ____________

15. 1.2934×10^{2} = ____________ 16. 2.69×10^{2} = ____________

17. 7.426×10^{4} = ____________ 18. 4.82×10^{3} = ____________

Topic Three: Square Roots

A. Definition of Square Root

Suppose we know the number of square feet in a square room, and we want to find the dimensions of the room. Since the sides of a square are equal, we use an operation called "square root" to find the measure of each of the equal sides. We can also use square roots for finding the length of a diagonal of a square and for finding the unknown side of a right triangle. (These types of problems will be explored in the next module Measurement.)

> The SQUARE ROOT of a power with an exponent of 2 is the base of that power. We use the symbol "$\sqrt{\ }$" (called the radical sign) to indicate a square root. The number under the radical sign is called the RADICAND.

EXAMPLE:

$\sqrt{56}$ (read "square root of fifty-six") has 56 as its radicand.

B. Square Root of a Perfect Square

You should recall that every perfect square can be written as a power with an exponent of 2. Thus we can easily find the square root of a perfect square.

EXAMPLE:

Find the square root of 144.
Solution

$$\sqrt{144} = \sqrt{(12)^2} = 12 \leftarrow$$

$144 = (12)^2$

The square root is the base of the power with the exponent of 2.

Complete the table below. You may refer to the table of perfect squares on pages 3 - 4.

Table of Square Roots of Perfect Squares from 0 to 625

PERFECT SQUARE	PERFECT SQUARE AS A POWER	SQUARE ROOT
0	0^2	$\sqrt{0^2} = 0$
1	1^2	$\sqrt{1^2} = 1$
9	3^2	$\sqrt{3^2} = 3$
25	5^2	$\sqrt{5^2} = 5$
121	11^2	$\sqrt{11^2} = 11$

PERFECT SQUARE	PERFECT SQUARE AS A POWER	SQUARE ROOT
225	15^2	$\sqrt{15^2} = 15$
289	17^2	$\sqrt{17^2} = 17$
400	20^2	$\sqrt{20^2} = 20$
	22^2	
625		

If we attach an even number of zeros to a perfect square the resulting number is a perfect square. (e.g., since 196 is a perfect square, 1960000 is also a perfect square.)

You recall that $1960000 = (1400)^2$ (Refer to the material on page 5.)

Thus $\sqrt{1960000} = \sqrt{(1400)^2} = 1400$

Notice there are half as many zeros in the square root as there are in the perfect square.

EXERCISE #3

Find the square root of each of the following:

1. $\sqrt{256}$ = 16 $\sqrt{25600}$ = 160 $\sqrt{256000000}$ = 16000
2. $\sqrt{169}$ = ______ $\sqrt{169000000}$ = ______ $\sqrt{16900000000}$ = ______
3. $\sqrt{529}$ = ______ $\sqrt{52900}$ = ______ $\sqrt{529000000}$ = ______

4. $\sqrt{361}$ = ______ $\sqrt{36100}$ = ______ $\sqrt{3610000}$ = ______

5. $\sqrt{4}$ = ______ $\sqrt{400}$ = ______ $\sqrt{400000000}$ = ______

C. Finding the Square Root of a Number Which is not a Perfect Square

Most numbers are not perfect squares, and we need to be able to find the square root of any number. A number which is not a perfect square cannot be written as a power with an exponent of two. Thus it does not have an exact square root; its square root is only approximate.

To find the approximate square root of a number, we find a power (with exponent 2) whose value is very close to the number. The base of the power is the approximate square root.

EXAMPLE:

Find the square root of 8 to three decimal places.

Solution

$(2.827)^2 = 7.991929$

$(2.828)^2 = 7.997584 \longleftarrow$ This is the closest to 8

$(2.829)^2 = 8.003241$

So $\sqrt{8} \doteq \sqrt{(2.828)^2} \doteq 2.828$ "is approximately equal to."

In this course, two methods of finding square roots are given. The second of these methods is optional.

I. Using a Square Root Table

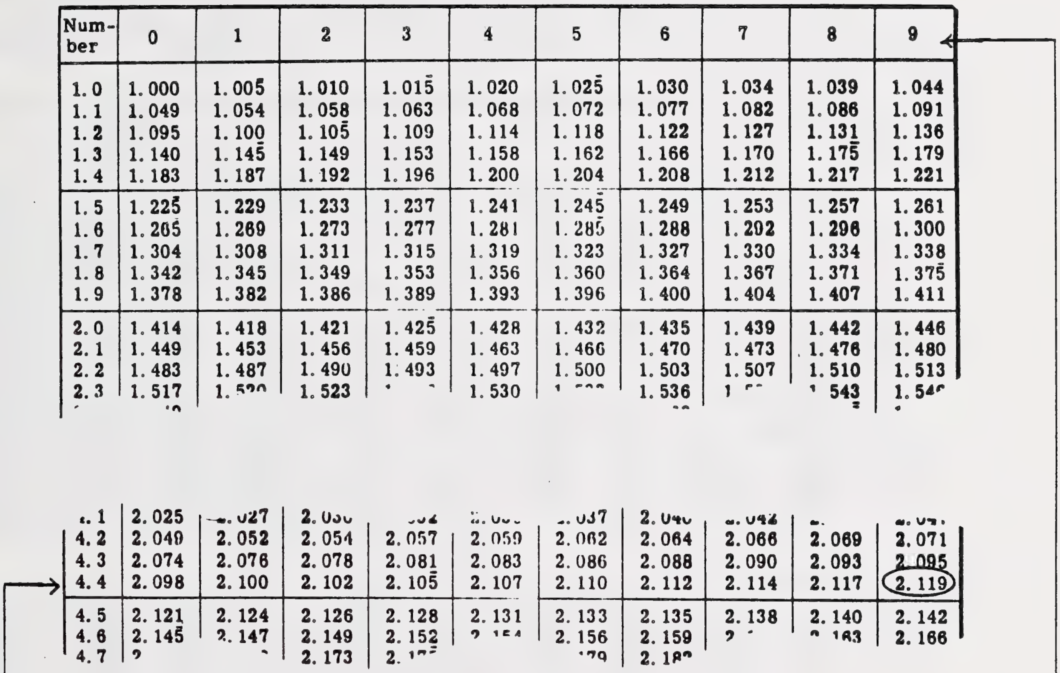
This method will be used most often, because it is the easier method. We use the tables on pages 26 - 29 for finding square roots.

EXAMPLES:

(a) Evaluate $\sqrt{4.49}$.

Solution

Since 4.49 is between 1.00 and 5.49, we look for the square root on page 26.



Number	0	1	2	3	4	5	6	7	8	9
1.0	1.000	1.005	1.010	1.01$\bar{5}$	1.020	1.02$\bar{5}$	1.030	1.034	1.039	1.044
1.1	1.049	1.054	1.058	1.063	1.068	1.072	1.077	1.082	1.086	1.091
1.2	1.095	1.10$\underline{0}$	1.10$\bar{5}$	1.109	1.114	1.118	1.122	1.127	1.13$\underline{1}$	1.136
1.3	1.140	1.14$\bar{5}$	1.149	1.153	1.158	1.162	1.166	1.170	1.17$\bar{5}$	1.179
1.4	1.183	1.187	1.192	1.196	1.200	1.204	1.208	1.212	1.217	1.221
1.5	1.22$\bar{5}$	1.229	1.233	1.237	1.241	1.245	1.249	1.253	1.257	1.261
1.6	1.26$\bar{5}$	1.269	1.273	1.277	1.281	1.28$\bar{5}$	1.288	1.202	1.296	1.300
1.7	1.304	1.308	1.311	1.315	1.319	1.323	1.327	1.330	1.334	1.338
1.8	1.342	1.345	1.349	1.353	1.356	1.360	1.364	1.367	1.371	1.37$\bar{5}$
1.9	1.378	1.382	1.386	1.389	1.393	1.396	1.400	1.404	1.407	1.411
2.0	1.414	1.418	1.421	1.42$\bar{5}$	1.428	1.432	1.435	1.439	1.442	1.446
2.1	1.449	1.453	1.456	1.459	1.463	1.466	1.470	1.473	1.476	1.480
2.2	1.483	1.487	1.490	1.493	1.497	1.500	1.503	1.507	1.510	1.513
2.3	1.517	[illegible]	1.523	[illegible]	1.530	[illegible]	1.536	[illegible]	543	[illegible]
[illegible]	2.025	.027	[illegible]	[illegible]	[illegible]	.037	[illegible]	[illegible]	[illegible]	[illegible]
4.2	2.049	2.052	2.054	2.057	2.059	2.062	2.064	2.066	2.069	2.071
4.3	2.074	2.076	2.078	2.08$\underline{1}$	2.083	2.086	2.088	2.090	2.093	2.095
4.4	2.098	2.100	2.102	2.10$\bar{5}$	2.107	2.110	2.112	2.114	2.117	2.119
4.5	2.12$\underline{1}$	2.124	2.126	2.128	2.131	2.133	2.135	2.138	2.140	2.142
4.6	2.14$\bar{5}$	2.147	2.149	2.15$\underline{2}$	[illegible]	2.156	2.159	[illegible]	[illegible]	2.166
4.7	[illegible]	[illegible]	2.173	[illegible]		[illegible]	[illegible]			

We read down the "Number" column until we come to 4.4. Then we look across that row in the column for 9. (i.e. 4.49)

Circle the entry in the 9's column in the row for 4.4.

Thus we find that $\sqrt{4.49} \doteq 2.119$.

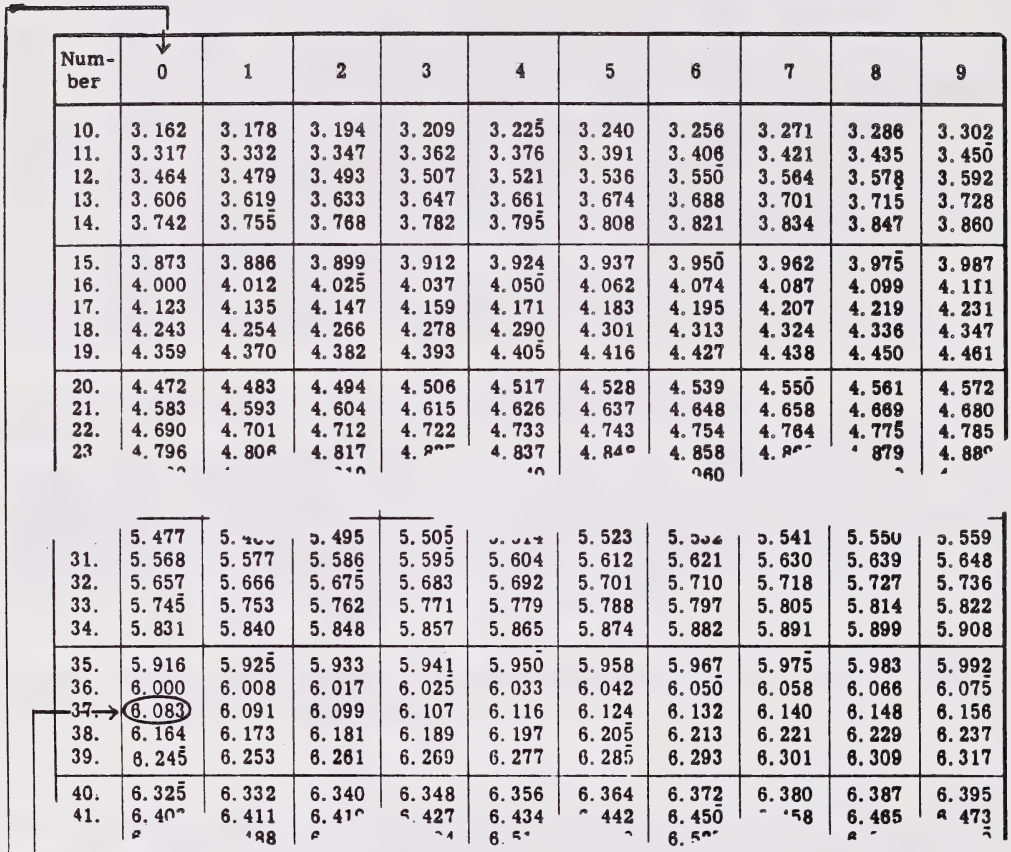
We then check our answer by squaring 2.119 to be sure that its value is very close to 4.49.

$$\begin{array}{r} 2.119 \\ \times\ \underline{2.119} \\ 19071 \\ 2119 \\ 2119 \\ \underline{4238} \\ 4.490161 \end{array} \longleftarrow \text{This is very close to 4.49.}$$

(b) Find $\sqrt{37}$. (37 has the same square root as 37.0.)

Solution

Since 37.0 is between 10.0 and 54.9, we find its square root on page 28.



Number	0	1	2	3	4	5	6	7	8	9
10.	3.162	3.178	3.194	3.209	3.22$\bar{5}$	3.240	3.256	3.271	3.286	3.30$\underline{2}$
11.	3.317	3.332	3.347	3.362	3.376	3.391	3.40$\underline{6}$	3.421	3.435	3.45$\bar{0}$
12.	3.464	3.479	3.493	3.507	3.521	3.536	3.55$\bar{0}$	3.564	3.57$\underline{8}$	3.592
13.	3.606	3.61$\underline{9}$	3.633	3.647	3.66$\underline{1}$	3.674	3.688	3.701	3.715	3.728
14.	3.742	3.75$\bar{5}$	3.768	3.782	3.79$\bar{5}$	3.808	3.821	3.834	3.847	3.860
15.	3.873	3.886	3.89$\underline{9}$	3.912	3.92$\underline{4}$	3.937	3.95$\bar{0}$	3.962	3.97$\bar{5}$	3.987
16.	4.000	4.012	4.02$\bar{5}$	4.037	4.05$\bar{0}$	4.062	4.074	4.087	4.099	4.111
17.	4.123	4.135	4.147	4.159	4.171	4.183	4.195	4.207	4.219	4.231
18.	4.243	4.254	4.266	4.278	4.29$\underline{0}$	4.301	4.313	4.324	4.336	4.347
19.	4.359	4.370	4.382	4.393	4.40$\bar{5}$	4.416	4.427	4.438	4.450	4.461
20.	4.472	4.483	4.494	4.506	4.517	4.528	4.539	4.55$\bar{0}$	4.561	4.572
21.	4.583	4.593	4.604	4.615	4.626	4.637	4.648	4.658	4.669	4.680
22.	4.690	4.701	4.712	4.722	4.733	4.743	4.754	4.764	4.77$\bar{5}$	4.785
23	4.796	4.80	4.817	4.	4.837	4.84	4.858	4.	879	4.88
							060			
	5.477	5.	5.495	5.50$\bar{5}$	[illegible]	5.523	5.	5.541	5.550	5.559
31.	5.568	5.577	5.58$\underline{6}$	5.59$\bar{5}$	5.604	5.612	5.621	5.630	5.639	5.648
32.	5.657	5.666	5.67$\bar{5}$	5.683	5.692	5.701	5.710	5.718	5.727	5.736
33.	5.74$\bar{5}$	5.753	5.762	5.771	5.779	5.788	5.797	5.805	5.814	5.822
34.	5.831	5.840	5.848	5.857	5.865	5.874	5.882	5.891	5.899	5.908
35.	5.916	5.92$\bar{5}$	5.933	5.94$\underline{1}$	5.95$\bar{0}$	5.958	5.96$\underline{7}$	5.97$\bar{5}$	5.983	5.99$\underline{2}$
36.	6.000	6.008	6.017	6.02$\bar{5}$	6.033	6.042	6.05$\bar{0}$	6.058	6.066	6.07$\bar{5}$
37.	6.083	6.091	6.099	6.107	6.116	6.12$\underline{4}$	6.132	6.140	6.148	6.156
38.	6.164	6.173	6.181	6.189	6.197	6.20$\bar{5}$	6.213	6.221	6.229	6.237
39.	6.24$\bar{5}$	6.253	6.261	6.269	6.277	6.28$\bar{5}$	6.293	6.301	6.309	6.317
40.	6.32$\bar{5}$	6.332	6.340	6.348	6.356	6.364	6.37$\underline{2}$	6.380	6.387	6.395
41.	6.40	6.411	6.41	.427	6.434	442	6.45$\bar{0}$	58	6.465	47$\underline{3}$
		488			6.5		6.5			

We read down the "Number" column until we come to 37. We read across that row in the 0's column.

Thus $\sqrt{37} \doteq$ __________.

Check:

$$\begin{array}{r} 6.083 \\ \times\ \underline{6.083} \\ 18249 \\ 48664 \\ \underline{364980} \\ 37.002889 \end{array}$$

← *This is very close to 37.*

EXERCISE #4

Using the tables on pages 26 - 29, find the square root for each of the following. (Check your answers for any three problems by squaring to see if the square is close to the original number.)

1. $\sqrt{12.8} \doteq$ 3.578

Check:

$$
\begin{array}{r}
3.578 \\
\times\ 3.578 \\
\hline
28624 \\
25046 \\
17890 \\
10734 \\
\hline
12.802084
\end{array}
$$

2. $\sqrt{19} \doteq$ ____________

3. $\sqrt{87.2} \doteq$ ____________

4. $\sqrt{2.3} \doteq$ ____________

5. $\sqrt{28.6} \doteq$ ____________

6. $\sqrt{9.87} \doteq$ ____________

7. $\sqrt{73} \doteq$ ____________

8. $\sqrt{59.0} \doteq$ ____________

(OPTIONAL SECTION)

II. Calculating Square Roots

This square root method is quite complex. It is to be learned by any students planning to apprentice in the electrical trades. Others may omit this section.

The procedure for calculating square roots involves two steps:

1. Separate the number into groups of two digits beginning at the decimal point.

2. Calculate the square root.

Let's practice separating some numbers into groups of 2 digits (beginning at the decimal point).

EXAMPLES:

(a) $\sqrt{1024}$ is rewritten as $\sqrt{10\ 24}$

(b) $\sqrt{103}$ is rewritten as $\sqrt{1\ 03}$

We could complete this group by placing a zero to the left of 1. Thus we could have 01 as the first group. Since 1 and 01 have the same value, the zero is seldom written.

(c) $\sqrt{1632.5954}$ is rewritten as $\sqrt{16\ 32\ .\ 59\ 54}$

(d) $\sqrt{159.324}$ is rewritten as $\sqrt{1\ 59\ .\ 32\ 40}$

The zero must be written to complete the last group of two digits. Since 4 and 40 do *not* have the same value, the zero cannot be omitted as it was in (b).

Self-Correcting Exercise #6

Separate the radicands into groups of two digits starting from the decimal point.

1. $\sqrt{135.9}$ ____________________

2. $\sqrt{163216}$ ____________________

3. $\sqrt{143}$ ____________________

4. $\sqrt{1764.29}$ ____________________

5. $\sqrt{1632.401}$ ____________________

Your answers should be checked with the solutions on page 25.

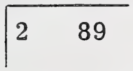
The easiest way to explain the method for calculating square roots is to give examples. In the following examples, the method will be carefully explained. Study these examples closely.

EXAMPLES:

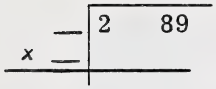
(a) Find $\sqrt{289}$

Solution

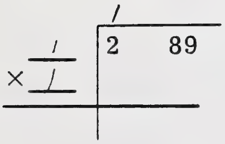
1. Separate 289 into groups of two digits, starting at the decimal point.



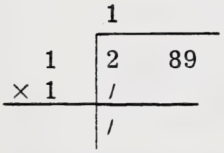
2. Calculate:



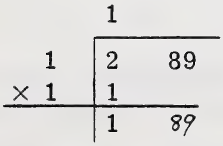
Place two blanks and a multiplication sign on the left as shown.



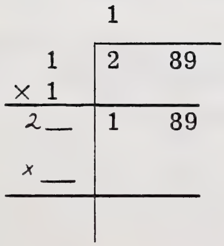
1 is the largest perfect square which is less than the value of the first group. The square root of 1 is 1. (i.e. $\sqrt{1} = \sqrt{1^2} = 1$). We place the square root, namely 1, above the first group and in each of the blanks on the left.



Place the product of the two numbers on the left below the first group and subtract (2 - 1 = 1).



Bring down the next group.



Double the number on top (2 × 1 = 2) and place the 2 with a blank to the left of 189. Also leave a blank for the multiplier.

Continued on page 18.

	1	9
1	2	89
× 1	1	
② 9	1	89
× 9		

Use 2 as a trial divisor into 18. The quotient is 9. Place 9 above the second group and in each of the blanks on the left.

	1	9
1	2	89
× 1		
29	1	89
× 9	2	61

Place the product of 29 × 9 = 261 below the 189. Since the product is too large, 9 is too large.

	1	8
1	2	89
× 1		
2 8	1	89
× 8	2	24

Next we try 8. 8 is also too large.

	1	7
1	2	89
× 1		
2 7	1	89
× 7	1	89

Finally we try 7. Since 27 × 7 = 189, there is no remainder.

Thus $\sqrt{289} = 17$

This answer could also be checked in your table of perfect squares.

Check:

$$\begin{array}{r} 17 \\ \times\ 17 \\ \hline 119 \\ 17 \\ \hline 289 \end{array}$$

(b) Calculate $\sqrt{1725.8}$ to the nearer tenth (i.e. to one decimal place.)

Solution

To calculate a square root to the nearer tenth, we find the square root to two decimal places, then round the answer to one decimal place.

Since each group of two digits represents only one digit in the square root, we must have two groups to the right of the decimal point.

1. Separate.

$$\overline{\big|\ 17 \quad 25 \ .\ 80 \quad 00}$$

We add one zero to complete the first group to the right of the decimal point. Two zeros are needed to form the second group.

2. Calculate.

```
      ________________________
   __ | 17  25  .  80  00
 × __ |
 -------------------------
      |
```

Place the blanks for the first grouping.

```
        4
      ________________________
    4 | 17  25  .  80  00
  × 4 |
 -------------------------
      |
```

16 is the largest perfect square which is less than the value of the first group. The square root of 16 is 4. (i.e. $\sqrt{16} = \sqrt{4^2} = 4$). We place the square root, namely 4, above the first group and in each of the blanks on the left.

```
        4
      ________________________
    4 | 17  25  .  80  00
  × 4 | 16
 -------------------------
      |  1
```

Place the product of the two numbers on the left below the first group and subtract (17 - 16 = 1).

```
        4
      ________________________
    4 | 17  25  .  80  00
  × 4 | 16
 -------------------------
      |  1  25
```

Bring down the next group.

```
        4
      ________________________
    4 | 17  25  .  80  00
  × 4 | 16
 -------------------------
  8 __|  1  25
  × __|
 -------------------------
      |
```

Double the number on top (2 × 4 = 8) and place the 8 with a blank to the left of 125. Also leave a blank for the multiplier.

```
        4   1
      ________________________
    4 | 17  25  .  80  00
  × 4 | 16
 -------------------------
  8 1 |  1  25
  × 1 |
 -------------------------
      |
```

Use 8 as a trial divisor into 12. The quotient is 1. Place 1 above the second group and in each of the blanks on the left.

```
        4   1
      ________________________
    4 | 17  25  .  80  00
  × 4 | 16
 -------------------------
   81 |  1  25
  × 1 |     81
 -------------------------
      |     44
```

Place the product of 81 × 1 = 81 below the 125 and subtract.

Continued on page 20.

```
           4    1  .
        ┌─────────────────────
     4  │ 17   25  .   80   00
   × 4  │ 16
  ──────┼─────────────────────
    81  │  1   25
   × 1  │      81
  ──────┼─────────────────────
        │      44      80
```

Place the decimal point in the answer above the decimal in the radicand. Bring down the next group.

```
           4    1  .
        ┌─────────────────────
     4  │ 17   25  .   80   00
   × 4  │ 16
  ──────┼─────────────────────
    81  │  1   25
   × 1  │      81
  ──────┼─────────────────────
   82__ │      44      80
    x__ │
  ──────┼─────────────────────
        │
```

Double the number on top (2 × 41 = 82) and place the 82 with a blank to the left of 4480. Also leave a blank for the multiplier

```
           4    1  .    5
        ┌─────────────────────
     4  │ 17   25  .   80   00
   × 4  │ 16
  ──────┼─────────────────────
    81  │  1   25
   × 1  │      81
  ──────┼─────────────────────
   825  │      44      80
   × 5  │
  ──────┼─────────────────────
        │
```

Use 82 as a trial divisor into 448. The quotient is 5. Place 5 above the third group and in each of the blanks on the left.

```
           4    1  .    5
        ┌─────────────────────
     4  │ 17   25  .   80   00
   × 4  │ 16
  ──────┼─────────────────────
    81  │  1   25
   × 1  │      81
  ──────┼─────────────────────
   825  │      44      80
   × 5  │      41      25
  ──────┼─────────────────────
        │       3      55
```

Place the product of 825 × 5 = 4125 below the 4480 and subtract.

```
           4    1  .    5
        ┌─────────────────────
     4  │ 17   25  .   80   00
   × 4  │ 16
  ──────┼─────────────────────
    81  │  1   25
   × 1  │      81
  ──────┼─────────────────────
   825  │      44      80
   × 5  │      41      25
  ──────┼─────────────────────
        │       3      55   00
```

Bring down the next group.

	4	1	.	5	
4	17	25	.	80	00
× 4	16				
81	1	25			
× 1		81			
825		44		80	
× 5		41		25	
830 __		3		55	00
× __					

Double the number on top (2 × 415 = 830) and place the 830 with a blank to the left of 35500. Also leave a blank for the multiplier.

	4	1	.	5	4
4	17	25	.	80	00
× 4	16				
81	1	25			
× 1		81			
825		44		80	
× 5		41		25	
8304		3		55	00
× 4					

Use 830 as a trial divisor into 3550. The quotient is 4. Place 4 above the fourth group and in each of the blanks on the left.

	4	1	.	5	4
4	17	25	.	80	00
× 4	16				
81	1	25			
× 1		81			
825		44		80	
× 5		41		25	
8304		3		55	00
× 4		3		32	16
				23	84

Place the product of 8304 × 4 = 33216 below the 35500 and subtract.

41.54 rounds to 41.5.

Thus $\sqrt{1725.8} \doteq 41.5$ to the nearer tenth.

Self-Correcting Exercise #7

Find each of the following square roots (to the nearer tenth).

1. $\sqrt{143.98} \doteq$ ____________ 2. $\sqrt{47.215} \doteq$ ____________

Calculation: Calculation:

Answers to these questions should be checked with the solutions on page 25.

EXERCISE #5

Find the square root of each of the following. (Find answers to the nearer tenth.) Show your work.

1. $\sqrt{362}$ 2. $\sqrt{133.26}$

Calculation: Calculation:

3. $\sqrt{68.263}$

Calculation:

4. $\sqrt{1226}$

Calculation:

Solutions to Self-Correcting Exercises

Self-Correcting Exercise #1, page 1

1. (a) 4^3 (b) a^5 (c) 7^2 (d) 12^4 (e) f^6 (f) r^9

2. (a) $2^7 = \underline{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$ (b) $6^2 = \underline{6 \times 6}$

(c) $q^3 = \underline{q \times q \times q}$ (d) $5^4 = \underline{5 \times 5 \times 5 \times 5}$

(e) $1^5 = \underline{1 \times 1 \times 1 \times 1 \times 1}$ (f) $t^{11} = \underline{t \times t \times t \times t \times t \times t \times t \times t \times t \times t \times t}$

Self-Correcting Exercise #2, page 2

1. $0^6 = (0 \times 0) \times (0 \times 0) \times (0 \times 0) = (0 \times 0) \times 0 = 0 \times 0 = \underline{0}$

2. $2^5 = (2 \times 2) \times (2 \times 2) \times 2 = (4 \times 4) \times 2 = 16 \times 2 = \underline{32}$

3. $7^2 = (7 \times 7) = \underline{49}$

4. $4^3 = (4 \times 4) \times 4 = (16 \times 4) = \underline{64}$

5. $6^4 = (6 \times 6) \times (6 \times 6) = (36 \times 36) = \underline{1296}$

Self-Correcting Exercise #3, page 5

3. 5^3 not a square
4. a^2 perfect square
5. 3^2 perfect square
6. 36 perfect square ($36 = 6^2$)
7. 225 perfect square ($225 = 15^2$)
8. 23^2 perfect square
9. 1903^5 not a square
10. 194^2 perfect square
11. 2^7 not a square
12. 169 perfect square ($169 = 13^2$)
13. 3^5 not a square
14. $16^2 = 16 \times 16 = 256$
15. $22^2 = 22 \times 22 = 484$
16. $19^2 = 19 \times 19 = 361$
17. 2^{15} not a square
18. $59^2 = 59 \times 59 = 3481$
19. 1398^{23} not a square
20. $40^2 = 40 \times 40 = 1600$
21. 47^7 not a square
22. $60^2 = 60 \times 60 = 3600$

Self-Correcting Exercise #4, page 6.

2. 2 zeros 3. 6 zeros 4. 2 zeros 5. 2 zeros 6. 8 zeros

7. $64 = (8)^2$, $6400 = (80)^2$, $640000 = (800)^2$

8. $9 = (3)^2$, $90000 = (300)^2$, $9000000 = (3000)^2$

9. $225 = (15)^2$, $22500 = (150)^2$, $2,250000 = (1500)^2$

10. $529 = (23)^2$, $52900 = (230)^2$, $529000000 = (23000)^2$

Self-Correcting Exercise #5, page 8

1. (a) 3.4×10^5 ✓ (b) 13.5×10^2 ______

(c) 2.0×10^8 ✓ (d) 7.1×10 ✓

(e) 0.1×10^3 ______ (f) 0.79×10^4 ______

(g) 6.23×10^4 ✓ (h) 0.439×10^2 ______

(i) 3.6×5^{10} ______ (j) 10×10^3 ______

2. (a) $156,000 = 1.56 \times 10^5$

(b) $97.3 = 9.73 \times 10^1$ or 9.73×10

(c) $728 = 7.28 \times 10^2$

(d) $2,500 = 2.5 \times 10^3$

(e) $1,687,000 = 1.687 \times 10^6$

(f) $7,235,000,000 = 7.235 \times 10^9$

(g) $1.3 = 1.3 \times 10^0$

(h) $1,090,000 = 1.09 \times 10^6$

(i) $465.2 = 4.652 \times 10^2$

(j) $24 = 2.4 \times 10^1$ or 2.4×10

3. (a) $7.77 \times 10^5 = 777,000$

(b) $34.8 \times 10^{13} = 348,000,000,000,000$

(c) $1.2 \times 10^8 = 120,000,000$

(d) $6.1 \times 10^4 = 61,000$

(e) $1.4325 \times 10^3 = 1,432.5$

(f) $2.8853 \times 16^6 = 2,885,300$

(g) $8.7056 \times 10^9 = 8,705,600,000$

(h) $5 \times 10^2 = 500$

(i) $4.05 \times 10^2 = 405$

(j) $8.2 \times 10 = 82$

(OPTIONAL SECTION)

Self-Correcting Exercise #6, page 16

1. $\sqrt{135.9} = \sqrt{1\ \ 35\ \ .\ \ 90}$
2. $\sqrt{163216} = \sqrt{16\ \ 32\ \ 16}$
3. $\sqrt{143} = \sqrt{1\ \ 43}$
4. $\sqrt{1764.29} = \sqrt{17\ \ 64\ \ .\ \ 29}$
5. $\sqrt{1632.401} = \sqrt{16\ \ 32\ \ .\ \ 40\ \ 10}$

Self-Correcting Exercise #7, page 22

1. $\sqrt{143.98} \doteq$ 12.0

	1 1 . 9 9
1	1 43 . 98 00
× 1	1
21	43
× 1	21
229	22 98
× 9	20 61
2389	2 37 00
× 9	2 15 01
	21 99

2. $\sqrt{47.215} \doteq$ 6.9

	6 . 8 7
6	47 . 21 50
× 6	36
128	11 21
× 8	10 24
1367	97 50
× 7	95 69
	1 81

SQUARE ROOTS OF NUMBERS 1.00 TO 5.49

Number	0	1	2	3	4	5	6	7	8	9
1.0	1.000	1.00$\bar{5}$	1.010	1.01$\bar{5}$	1.020	1.02$\bar{5}$	1.030	1.034	1.039	1.044
1.1	1.049	1.054	1.058	1.063	1.068	1.072	1.077	1.082	1.086	1.091
1.2	1.095	1.100	1.10$\bar{5}$	1.109	1.114	1.118	1.122	1.127	1.131	1.136
1.3	1.140	1.14$\bar{5}$	1.149	1.153	1.158	1.162	1.166	1.170	1.17$\bar{5}$	1.179
1.4	1.183	1.187	1.192	1.196	1.200	1.204	1.208	1.212	1.217	1.221
1.5	1.22$\bar{5}$	1.229	1.233	1.237	1.241	1.24$\bar{5}$	1.249	1.253	1.257	1.261
1.6	1.26$\bar{5}$	1.269	1.273	1.277	1.281	1.28$\bar{5}$	1.288	1.292	1.296	1.300
1.7	1.304	1.308	1.311	1.315	1.319	1.323	1.327	1.330	1.334	1.338
1.8	1.342	1.345	1.349	1.353	1.356	1.360	1.364	1.367	1.371	1.37$\bar{5}$
1.9	1.378	1.382	1.386	1.389	1.393	1.396	1.400	1.404	1.407	1.411
2.0	1.414	1.418	1.421	1.42$\bar{5}$	1.428	1.432	1.435	1.439	1.442	1.446
2.1	1.449	1.453	1.456	1.459	1.463	1.466	1.470	1.473	1.476	1.480
2.2	1.483	1.487	1.490	1.493	1.497	1.500	1.503	1.507	1.510	1.513
2.3	1.517	1.520	1.523	1.526	1.530	1.533	1.536	1.539	1.543	1.546
2.4	1.549	1.552	1.556	1.559	1.562	1.565	1.568	1.572	1.57$\bar{5}$	1.578
2.5	1.581	1.584	1.587	1.591	1.594	1.597	1.600	1.603	1.606	1.609
2.6	1.612	1.616	1.619	1.622	1.62$\bar{5}$	1.628	1.631	1.634	1.637	1.640
2.7	1.643	1.646	1.649	1.652	1.655	1.658	1.661	1.664	1.667	1.670
2.8	1.673	1.676	1.679	1.682	1.685	1.688	1.691	1.694	1.697	1.700
2.9	1.703	1.706	1.709	1.712	1.71$\bar{5}$	1.718	1.720	1.723	1.726	1.729
3.0	1.732	1.73$\bar{5}$	1.738	1.741	1.744	1.746	1.749	1.752	1.75$\bar{5}$	1.758
3.1	1.761	1.764	1.766	1.769	1.772	1.77$\bar{5}$	1.778	1.780	1.783	1.786
3.2	1.789	1.792	1.794	1.797	1.800	1.803	1.806	1.808	1.811	1.814
3.3	1.817	1.819	1.822	1.82$\bar{5}$	1.828	1.830	1.833	1.836	1.838	1.841
3.4	1.844	1.847	1.849	1.852	1.85$\bar{5}$	1.857	1.860	1.863	1.865	1.868
3.5	1.871	1.873	1.876	1.879	1.881	1.884	1.887	1.889	1.892	1.89$\bar{5}$
3.6	1.897	1.900	1.903	1.905	1.908	1.910	1.913	1.916	1.918	1.921
3.7	1.924	1.926	1.929	1.931	1.934	1.936	1.939	1.942	1.944	1.947
3.8	1.949	1.952	1.954	1.957	1.960	1.962	1.96$\bar{5}$	1.967	1.970	1.972
3.9	1.97$\bar{5}$	1.977	1.980	1.982	1.98$\bar{5}$	1.987	1.990	1.992	1.99$\bar{5}$	1.997
4.0	2.000	2.002	2.00$\bar{5}$	2.007	2.010	2.012	2.01$\bar{5}$	2.017	2.020	2.022
4.1	2.02$\bar{5}$	2.027	2.030	2.032	2.03$\bar{5}$	2.037	2.040	2.042	2.04$\bar{5}$	2.047
4.2	2.049	2.052	2.054	2.057	2.059	2.062	2.064	2.066	2.069	2.071
4.3	2.074	2.076	2.078	2.081	2.083	2.086	2.088	2.090	2.093	2.095
4.4	2.098	2.100	2.102	2.10$\bar{5}$	2.107	2.110	2.112	2.114	2.117	2.119
4.5	2.121	2.124	2.126	2.128	2.131	2.133	2.135	2.138	2.140	2.142
4.6	2.14$\bar{5}$	2.147	2.149	2.152	2.154	2.156	2.159	2.161	2.163	2.166
4.7	2.168	2.170	2.173	2.17$\bar{5}$	2.177	2.179	2.182	2.184	2.186	2.189
4.8	2.191	2.193	2.195	2.198	2.200	2.202	2.20$\bar{5}$	2.207	2.209	2.211
4.9	2.214	2.216	2.218	2.220	2.223	2.22$\bar{5}$	2.227	2.229	2.232	2.234
5.0	2.236	2.238	2.241	2.243	2.24$\bar{5}$	2.247	2.249	2.252	2.254	2.256
5.1	2.258	2.261	2.263	2.26$\bar{5}$	2.267	2.269	2.272	2.274	2.276	2.278
5.2	2.280	2.283	2.28$\bar{5}$	2.287	2.289	2.291	2.293	2.296	2.298	2.300
5.3	2.302	2.304	2.307	2.309	2.311	2.313	2.315	2.317	2.319	2.322
5.4	2.324	2.326	2.328	2.330	2.332	2.33$\bar{5}$	2.337	2.339	2.341	2.343
	0	1	2	3	4	5	6	7	8	9

SQUARE ROOTS OF NUMBERS 5.50 TO 9.99

Number	0	1	2	3	4	5	6	7	8	9
5.5	2.345	2.347	2.349	2.352	2.354	2.356	2.358	2.360	2.362	2.364
5.6	2.366	2.369	2.371	2.373	2.375	2.377	2.379	2.381	2.383	2.385
5.7	2.387	2.390	2.392	2.394	2.396	2.398	2.400	2.402	2.404	2.406
5.8	2.408	2.410	2.412	2.415	2.417	2.419	2.421	2.423	2.425	2.427
5.9	2.429	2.431	2.433	2.435	2.437	2.439	2.441	2.443	2.445	2.447
6.0	2.449	2.452	2.454	2.456	2.458	2.460	2.462	2.464	2.466	2.468
6.1	2.470	2.472	2.474	2.476	2.478	2.480	2.482	2.484	2.486	2.488
6.2	2.490	2.492	2.494	2.496	2.498	2.500	2.502	2.504	2.506	2.508
6.3	2.510	2.512	2.514	2.516	2.518	2.520	2.522	2.524	2.526	2.528
6.4	2.530	2.532	2.534	2.536	2.538	2.540	2.542	2.544	2.546	2.548
6.5	2.550	2.551	2.553	2.555	2.557	2.559	2.561	2.563	2.565	2.567
6.6	2.569	2.571	2.573	2.575	2.577	2.579	2.581	2.583	2.585	2.587
6.7	2.588	2.590	2.592	2.594	2.596	2.598	2.600	2.602	2.604	2.606
6.8	2.608	2.610	2.612	2.613	2.615	2.617	2.619	2.621	2.623	2.625
6.9	2.627	2.629	2.631	2.632	2.634	2.636	2.638	2.640	2.642	2.644
7.0	2.646	2.648	2.650	2.651	2.653	2.655	2.657	2.659	2.661	2.663
7.1	2.665	2.666	2.668	2.670	2.672	2.674	2.676	2.678	2.680	2.681
7.2	2.683	2.685	2.687	2.689	2.691	2.693	2.694	2.696	2.698	2.700
7.3	2.702	2.704	2.706	2.707	2.709	2.711	2.713	2.715	2.717	2.718
7.4	2.720	2.722	2.724	2.726	2.728	2.729	2.731	2.733	2.735	2.737
7.5	2.739	2.740	2.742	2.744	2.746	2.748	2.750	2.751	2.753	2.755
7.6	2.757	2.759	2.760	2.762	2.764	2.766	2.768	2.769	2.771	2.773
7.7	2.775	2.777	2.778	2.780	2.782	2.784	2.786	2.787	2.789	2.791
7.8	2.793	2.795	2.796	2.798	2.800	2.802	2.804	2.805	2.807	2.809
7.9	2.811	2.812	2.814	2.816	2.818	2.820	2.821	2.823	2.825	2.827
8.0	2.828	2.830	2.832	2.834	2.835	2.837	2.839	2.841	2.843	2.844
8.1	2.846	2.848	2.850	2.851	2.853	2.855	2.857	2.858	2.860	2.862
8.2	2.864	2.865	2.867	2.869	2.871	2.872	2.874	2.876	2.877	2.879
8.3	2.881	2.883	2.884	2.886	2.888	2.890	2.891	2.893	2.895	2.897
8.4	2.898	2.900	2.902	2.903	2.905	2.907	2.909	2.910	2.912	2.914
8.5	2.915	2.917	2.919	2.921	2.922	2.924	2.926	2.927	2.929	2.931
8.6	2.933	2.934	2.936	2.938	2.939	2.941	2.943	2.944	2.946	2.948
8.7	2.950	2.951	2.953	2.955	2.956	2.958	2.960	2.961	2.963	2.965
8.8	2.966	2.968	2.970	2.972	2.973	2.975	2.977	2.978	2.980	2.982
8.9	2.983	2.985	2.987	2.988	2.990	2.992	2.993	2.995	2.997	2.998
9.0	3.000	3.002	3.003	3.005	3.007	3.008	3.010	3.012	3.013	2.015
9.1	3.017	3.018	3.020	3.022	3.023	3.025	3.027	3.028	2.030	3.032
9.2	3.033	3.035	3.036	3.038	3.040	3.041	3.043	3.045	3.046	3.048
9.3	3.050	3.051	3.053	3.055	3.056	3.058	3.059	3.061	3.063	3.064
9.4	3.066	3.068	3.069	3.071	3.072	3.074	3.076	3.077	3.079	3.081
9.5	3.082	3.084	3.085	3.087	3.089	3.090	3.092	3.094	3.095	3.097
9.6	3.098	3.100	3.102	3.103	3.105	3.106	3.108	3.110	3.111	3.113
9.7	3.114	3.116	3.118	3.119	3.121	3.122	3.124	3.126	3.127	3.129
9.8	3.130	3.132	3.134	3.135	3.137	3.138	3.140	3.142	3.143	3.145
9.9	3.146	3.148	3.150	3.151	3.153	3.154	3.156	3.158	3.159	3.161
	0	1	2	3	4	5	6	7	8	9

SQUARE ROOTS OF NUMBERS 10.0 TO 54.9

Number	0	1	2	3	4	5	6	7	8	9
10.	3.162	3.178	3.194	3.209	3.225	3.240	3.256	3.271	3.286	3.302
11.	3.317	3.332	3.347	3.362	3.376	3.391	3.406	3.421	3.435	3.450
12.	3.464	3.479	3.493	3.507	3.521	3.536	3.550	3.564	3.578	3.592
13.	3.606	3.619	3.633	3.647	3.661	3.674	3.688	3.701	3.715	3.728
14.	3.742	3.755	3.768	3.782	3.795	3.808	3.821	3.834	3.847	3.860
15.	3.873	3.886	3.899	3.912	3.924	3.937	3.950	3.962	3.975	3.987
16.	4.000	4.012	4.025	4.037	4.050	4.062	4.074	4.087	4.099	4.111
17.	4.123	4.135	4.147	4.159	4.171	4.183	4.195	4.207	4.219	4.231
18.	4.243	4.254	4.266	4.278	4.290	4.301	4.313	4.324	4.336	4.347
19.	4.359	4.370	4.382	4.393	4.405	4.416	4.427	4.438	4.450	4.461
20.	4.472	4.483	4.494	4.506	4.517	4.528	4.539	4.550	4.561	4.572
21.	4.583	4.593	4.604	4.615	4.626	4.637	4.648	4.658	4.669	4.680
22.	4.690	4.701	4.712	4.722	4.733	4.743	4.754	4.764	4.775	4.785
23.	4.796	4.806	4.817	4.827	4.837	4.848	4.858	4.868	4.879	4.889
24.	4.899	4.909	4.919	4.930	4.940	4.950	4.960	4.970	4.980	4.990
25.	5.000	5.010	5.020	5.030	5.040	5.050	5.060	5.070	5.079	5.089
26.	5.099	5.109	5.119	5.128	5.138	5.148	5.158	5.167	5.177	5.187
27.	5.196	5.206	5.215	5.225	5.235	5.244	5.254	5.263	2.273	5.282
28.	5.292	5.301	5.310	5.320	5.329	5.339	5.348	5.357	5.367	5.376
29.	5.385	5.394	5.404	5.413	5.422	5.431	5.441	5.450	5.459	5.468
30.	5.477	5.486	5.495	5.505	5.514	5.523	5.532	5.541	5.550	5.559
31.	5.568	5.577	5.586	5.595	5.604	5.612	5.621	5.630	5.639	5.648
32.	5.657	5.666	5.675	5.683	5.692	5.701	5.710	5.718	5.727	5.736
33.	5.745	5.753	5.762	5.771	5.779	5.788	5.797	5.805	5.814	5.822
34.	5.831	5.840	5.848	5.857	5.865	5.874	5.882	5.891	5.899	5.908
35.	5.916	5.925	5.933	5.941	5.950	5.958	5.967	5.975	5.983	5.992
36.	6.000	6.008	6.017	6.025	6.033	6.042	6.050	6.058	6.066	6.075
37.	6.083	6.091	6.099	6.107	6.116	6.124	6.132	6.140	6.148	6.156
38.	6.164	6.173	6.181	6.189	6.197	6.205	6.213	6.221	6.229	6.237
39.	6.245	6.253	6.261	6.269	6.277	6.285	6.293	6.301	6.309	6.317
40.	6.325	6.332	6.340	6.348	6.356	6.364	6.372	6.380	6.387	6.395
41.	6.403	6.411	6.419	6.427	6.434	6.442	6.450	6.458	6.465	6.473
42.	6.481	6.488	6.496	6.504	6.512	6.519	6.527	6.535	6.542	6.550
43.	6.557	6.565	6.573	6.580	6.588	6.595	6.603	6.611	6.618	6.626
44.	6.633	6.641	6.648	6.656	6.663	6.671	6.678	6.686	6.693	6.701
45.	6.708	6.716	6.723	6.731	6.738	6.745	6.753	6.760	6.768	6.775
46.	6.782	6.790	6.797	6.804	6.812	6.819	6.826	6.834	6.841	6.848
47.	6.856	6.863	6.870	6.877	6.885	6.892	6.899	6.907	6.914	6.921
48.	6.928	6.935	6.943	6.950	6.957	6.964	6.971	6.979	6.986	6.993
49.	7.000	7.007	7.014	7.021	7.029	7.036	7.043	7.050	7.057	7.064
50.	7.071	7.078	7.085	7.092	7.099	7.106	7.113	7.120	7.127	7.134
51.	7.141	7.148	7.155	7.162	7.169	7.176	7.183	7.190	7.197	7.204
52.	7.211	7.218	7.225	7.232	7.239	7.246	7.253	7.259	7.266	7.273
53.	7.280	7.287	7.294	7.301	7.308	7.314	7.321	7.328	7.335	7.342
54.	7.348	7.355	7.362	7.369	7.376	7.382	7.389	7.396	7.403	7.409
	0	1	2	3	4	5	6	7	8	9

SQUARE ROOTS OF NUMBERS 55.0 TO 99.9

Numbers	0	1	2	3	4	5	6	7	8	9
55.	7.416	7.423	7.430	7.436	7.443	7.450̅	7.457	7.463	7.470	7.477
56.	7.483	7.490	7.497	7.503	7.510	7.517	7.523	7.530	7.537	7.543
57.	7.550̅	7.556	7.563	7.570	7.576	7.583	7.589	7.596	7.603	7.609
58.	7.616	7.622	7.629	7.635	7.642	7.649	7.655	7.662	7.668	7.675̅
59.	7.681	7.688	7.694	7.701	7.707	7.714	7.720	7.727	7.733	7.740
60.	7.746	7.752	7.759	7.765	7.772	7.778	7.785̅	7.791	7.797	7.804
61.	7.810	7.817	7.823	7.829	7.836	7.842	7.849	7.855̅	7.861	7.868
62.	7.874	7.880	7.887	7.893	7.899	7.906	7.912	7.918	7.925̅	7.931
63.	7.937	7.944	7.950̅	7.956	7.962	7.969	7.975̅	7.981	7.987	7.994
64.	8.000	8.006	8.012	8.019	8.025̅	8.031	8.037	8.044	8.050̅	8.056
65.	8.062	8.068	8.075̅	8.081	8.087	8.093	8.099	8.106	8.112	8.118
66.	8.124	8.130	8.136	8.142	8.149	8.155̅	8.161	8.167	8.173	8.179
67.	8.185	8.191	8.198	8.204	8.210	8.216	8.222	8.228	8.234	8.240
68.	8.246	8.252	8.258	8.264	8.270	8.276	8.283	8.289	8.295̅	8.301
69.	8.307	8.313	8.319	8.325̅	8.331	8.337	8.343	8.349	8.355̅	8.361
70.	8.367	8.373	8.379	8.385	8.390	8.396	8.402	8.408	8.414	8.420
71.	8.426	8.432	8.438	8.444	8.450̅	8.456	8.462	8.468	8.473	8.479
72.	8.485	8.491	8.497	8.503	8.509	8.515̅	8.521	8.526	8.532	8.538
73.	8.544	8.550̅	8.556	8.562	8.567	8.573	8.579	8.585̅	8.591	8.597
74.	8.602	8.608	8.614	8.620	8.626	8.631	8.637	8.643	8.649	8.654
75.	8.660	8.666	8.672	8.678	8.683	8.689	8.695̅	8.701	8.706	8.712
76.	8.718	8.724	8.729	8.735̅	8.741	8.746	8.752	8.758	8.764	8.769
77.	8.775̅	8.781	8.786	8.792	8.798	8.803	8.809	8.815̅	8.820	8.826
78.	8.832	8.837	8.843	8.849	8.854	8.860	8.866	8.871	8.877	8.883
79.	8.888	8.894	8.899	8.905	8.911	8.916	8.922	8.927	8.933	8.939
80.	8.944	8.950̅	8.955	8.961	8.967	8.972	8.978	8.983	8.989	8.994
81.	9.000	9.006	9.011	9.017	9.022	9.028	9.033	9.039	9.044	9.050̅
82.	9.055	9.061	9.066	9.072	9.077	9.083	9.088	9.094	9.099	9.105̅
83.	9.110	9.116	9.121	9.127	9.132	9.138	9.143	9.149	9.154	9.160
84.	9.165	9.171	9.176	9.182	9.187	9.192	9.198	9.203	9.209	9.214
85.	9.220	9.225̅	9.230	9.236	9.241	9.247	9.252	9.257	9.263	9.268
86.	9.274	9.279	9.284	9.290	9.295	9.301	9.306	9.311	9.317	9.322
87.	9.327	9.333	9.338	9.343	9.349	9.354	9.359	9.365̅	9.370	9.375
88.	9.381	9.386	9.391	9.397	9.402	9.407	9.413	9.418	9.423	9.429
89.	9.434	9.439	9.445̅	9.450̅	9.455	9.460	9.466	9.471	9.476	9.482
90.	9.487	9.492	9.497	9.503	9.508	9.513	9.518	9.524	9.529	9.534
91.	9.539	9.545̅	9.550̅	9.555	9.560	9.566	9.571	9.576	9.581	9.586
92.	9.592	9.597	9.602	9.607	9.612	9.618	9.623	9.628	9.633	9.638
93.	9.644	9.649	9.654	9.659	9.664	9.670	9.675̅	9.680	9.685	9.690
94.	9.695	9.701	9.706	9.711	9.716	9.721	9.726	9.731	9.737	9.742
95.	9.747	9.752	9.757	9.762	9.767	9.772	9.778	9.783	9.788	9.793
96.	9.798	9.803	9.808	9.813	9.818	9.823	9.829	9.834	9.839	9.844
97.	9.849	9.854	9.859	9.864	9.869	9.874	9.879	9.884	9.889	9.894
98.	9.899	9.905̅	9.910	9.915̅	9.920	9.925̅	9.930	9.935̅	9.940	9.945̅
99.	9.950̅	9.955̅	9.960	9.965̅	9.970	9.975̅	9.980	9.985̅	9.990	9.995̅
	0	1	2	3	4	5	6	7	8	9

End of Lesson 6

Practical Mathematics
Module 4
Answer Keys

PART A

Page 1

1. (a) hundreds; (3×100)

   tenths; $(7 \times \frac{1}{10})$

   (b) thousands; (8×1000)

   thousandths; $(9 \times \frac{1}{1000}$

   (c) ten-thousandths; $(2 \times \frac{1}{10\ 000}$

   hundred-thousandths; $(6 \times \frac{1}{100\ 000}$

2. (a)
   (b) 7 030
   (c) 5 236.375
   (d) 80 305
   (e) 75 035.462

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   (f) 10 000.000 1
   (g) 7.003

3. (a) 0.409
   (b) 100 017
   (c) 4 765 209.06
   (d) 10 400
   (e) 0.0145

4. (a) E (b) E
   (c) N (d) N
   (e) E (f) N

5. (a) $6\frac{2}{5}$

   (b) $17\frac{61}{100}$

   (c) $2\frac{5}{7}$

   (d) $919\frac{7154}{10\ 000} = 919\frac{3577}{5000}$

   (e) $\frac{164}{13} = 12\frac{8}{13}$

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   (f) $6\frac{625}{1000} = 6\frac{25}{40} = 6\frac{5}{8}$

   (g) $40\frac{352}{10\ 000} = 40\frac{22}{625}$

   (h) $5\frac{21\ 746}{100\ 000} = 5\frac{10\ 873}{50\ 000}$

   (i) $81\frac{558}{1000} = 81\frac{279}{500}$

   (j) $\frac{1001}{4} = 250\frac{1}{4}$

6. (a) <
   (b) <
   (c) >
   (d) >
   (e) >
   (f) <
   (g) =
   (h) =
   (i) <
   (j) <

7. (a) 50.7
   (b) 132 900
   (c) 0.058 32
   (d) 37.96
   (e) 1 741 000
   (f) 832 070
   (g) 0.002 9325
   (h) 0.000 393 42
   (i) 0.000 107
   (j) 900

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8. (a) 0.05 (b) 0.0035
(c) 0.0011 (d) 0.00225
(e) 0.025 (f) 2.9
(g) 0.98 (h) 5.572
(i) 0.3 (j) 40

9. (a) $\frac{9}{125}$ (b) $\frac{7}{100}$

(c) $\frac{143}{1000}$ (d) $\frac{32}{100} = \frac{8}{25}$

(e) $2\frac{5}{10} = 2\frac{1}{2}$ (f) $\frac{103}{100} = 1\frac{3}{100}$

(g) $\frac{9}{10}$ (h) $6\frac{41}{1000}$

(i) $12\frac{75}{100} = 12\frac{3}{4}$ (j) $1\frac{75}{100} = 1\frac{3}{4}$

10. (a) 100
(b) 1.3
(c) 1300
(d) 69
(e) 0.23
(f) 590
(g) 1.0
(h) 1 532 000
(i) 1.37
(j) 6.954

PART B

Page 5

1. (a) 148.472
(b) 27.464
(c) 941.750 1
(d) 19.39

(a)	(b) 0.039 17.500 9.250 0.675 27.464	(c) 2.6831 98.5670 840.5000 941.7501	(d) 0.34 0.80 18.00 0.25 19.39

2. (a) 0.996 (b) 9.066
(c) 14.606 (d) 37.57
(e) 141.013

3. (a) 537.6 (b) 30.900
(c) 0.053 76 (d) 1.911
(e) 0.00028 (f) 0.020 577

4. (a) 0.0272 (b) 8.28
(c) 13.80 (d) 1.36
(e) 1.26 (f) 0.105

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5. (a)

(b) = **$916.79**

$$\begin{array}{r} 985.8 \\ 0.93 \\ \hline 29574 \\ 88722 \\ \hline 916.794 \end{array}$$

(c) = **$2097.94**

$$\begin{array}{r} 30852 \\ \times 0.068 \\ \hline 246816 \\ 185112 \\ \hline 2097.936 \end{array}$$

6. (a) 2.89

2.89
17)49.13
34
151
136
153
153
0

(b) 48

48
.24)11.52
96
192
192
0

(c) 0.38

0.38
1.9).722
57
152
152
0

(d) 0.529

0.529
23)12.167
115
66
46
207
207
0

(e) 338

338
.013)4.394
39
49
39
104
104
0

(f) 720

720
.726)522.720
5082
1452
1452
0
0
0

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7. (a)

(b) $23.10

23.10
1.8)41.58
36
55
54
18
18
0

(c) \$42.77

$$\begin{array}{r}
42.773 \\
.2553\overline{)10.9200000} \\
\underline{10212} \\
7080 \\
\underline{5106} \\
19740 \\
\underline{17871} \\
18690 \\
\underline{17871} \\
8190 \\
\underline{7654} \\
531
\end{array}$$

(d) \$340.12

$$\begin{array}{r}
340.119 \\
.167\overline{)56.800000} \\
\underline{501} \\
670 \\
\underline{668} \\
200 \\
\underline{167} \\
330 \\
\underline{167} \\
1630 \\
\underline{1503} \\
127
\end{array}$$

8. (a)

(b) $1.8\overline{3}$

$$\begin{array}{r}
1.833 \\
6\overline{)11.000} \\
\underline{6} \\
50 \\
\underline{48} \\
20 \\
\underline{18} \\
20 \\
\underline{18} \\
2
\end{array}$$

Page 8

(c) 0.5625

$$\begin{array}{r}
.5625 \\
16\overline{)9.0000} \\
\underline{80} \\
100 \\
\underline{96} \\
40 \\
\underline{32} \\
80 \\
\underline{80} \\
0
\end{array}$$

(d) $0.41\overline{6}$

$$\begin{array}{r}
.4166 \\
12\overline{)5.0000} \\
\underline{48} \\
20 \\
\underline{12} \\
80 \\
\underline{72} \\
80 \\
\underline{72} \\
8
\end{array}$$

(e) $0.3\overline{8}$

.388
18)7.00
54
160
144
160
144
16

(f) 1.12

1.12
25)28.000
25
30
25
50
50
0

9. (a)

3.323
71)236.000
213
230
213
170
142
280
213
67

236 ÷ 71 = 3.32

(b)

905
×509
8145
45250
460645

(c)

7001
−309
6692

(d) 36.09 − 21.6 = **14.49**

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(e)

16.325
×0.68
130600
97950
11.10100

(f) 123 × 1.96 = **241.08**

123
1.96
738
1107
123
241.08

(g) $3.195 \div 1.2 = 2.7$

$$\begin{array}{r}
2.66 \\
1.2\overline{)3.195} \\
\underline{24} \\
79 \\
\underline{72} \\
75 \\
\underline{72} \\
3
\end{array}$$

(h) $\frac{1}{2} + \frac{5}{6} + \frac{1}{3} = \frac{5}{3} = 1\frac{2}{3}$

$\frac{3}{6} + \frac{5}{6} + \frac{2}{6} = \frac{10}{6} = \frac{5}{3} = 1\frac{2}{3}$

(i) $\frac{4}{7}$

$\frac{1}{4} \times \frac{16}{7} = \frac{4}{7}$

(j) $\frac{5}{8}$

$$\begin{array}{r}
3 = 2\frac{8}{8} \\
\underline{-2\frac{3}{8} = -2\frac{3}{8}} \\
\frac{5}{8}
\end{array}$$

(k) $\frac{117}{16} = 7\frac{5}{16}$

$1\frac{5}{8} \times 9 \times \frac{1}{2}$

$= \frac{13}{8} \times \frac{9}{1} \times \frac{1}{2} = \frac{117}{16} = 7\frac{5}{16}$

(l) $1.356 \div 10\,000$
$= 0.001356$

10. (a)

(b)
$$\begin{array}{r}
785 \\
290 \\
107 \\
44 \\
\underline{6} \\
1232
\end{array}$$

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(c)
$$\begin{array}{r} 1.090 \\ 0.152 \\ \underline{0.800} \\ 1.042 \end{array}$$

(d)
$$\begin{array}{r} 700.033 \\ \underline{359.650} \\ 1059.683 \end{array}$$
[The word and means decimal]

(e)
$$\begin{array}{r} 5.6000 \\ \underline{-0.0301} \\ 5.5699 \end{array}$$

(f)
$$\begin{array}{rcl} 10\frac{1}{3} = 10\frac{8}{24} & = & 9\frac{32}{24} \\ -5\frac{3}{8} = 5\frac{9}{24} & = & 5\frac{9}{24} \\ \hline & = & 4\frac{23}{24} \end{array}$$

PART C

1. 5.5 ÷ 1.25

$$\begin{array}{r} 4.4 \\ 1.25\overline{)5.500} \\ \underline{500} \\ 500 \\ \underline{500} \\ 0 \end{array}$$

4 shelves can be cut from a 5.5 metre board.

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2. 225 × 12 = 2700 (225 feet = 2700 inches)

$$\begin{array}{r} 490 \\ 5.5\overline{)2700.0} \\ \underline{250} \\ 200 \\ \underline{495} \\ 50 \end{array}$$

490 pieces of wire 5.5 inches long can be cut from a piece 225 feet long.

3. 0.12'' × 50
= 6''

In 50 feet the pipe will drop 6 inches.

4. 292.85 ÷ 32.3
= 9.06656...
= 9.07 or 9.1 km

$$\begin{array}{r} 9.06 \\ 32.3\overline{)292.85} \\ \underline{2907} \\ 215 \\ \underline{0} \\ 2150 \\ \underline{1938} \\ 212 \end{array}$$

The new model travels 9.1 km on each litre of gasoline.

5. Cost = 52.5 × $9.25
= 485.625
= $485.63

The floor covering costs $485.63.

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6. No. of kg $= 1.46 \times 1000$
$= 1460$ kg

$$\text{Cost} = 1460 \times \frac{\$19.26}{100}$$
$$= 14.6 \times 19.26$$
$$= 281.196$$
$$= \$281.20$$

The steel rod costs $281.20.

PART D

1. (a) $= \frac{59}{7} \times \frac{35}{181} = \frac{295}{181} = 1\frac{114}{181}$

(b) $= \dfrac{\frac{10}{12} + \frac{9}{12}}{\frac{1}{2}} = \frac{19}{12} \times \frac{2}{1} = \frac{19}{6} = 3\frac{1}{6}$

(c) $\dfrac{5 + 2}{\frac{4}{6} - \frac{3}{6}} = \dfrac{7}{\frac{1}{6}} = 7 \times \frac{6}{1} = 42$

(d) $= \dfrac{9}{5 \times \frac{5}{2}} = \dfrac{9}{\frac{25}{2}} = 9 \times \frac{2}{25} = \frac{18}{25}$

2. (a)
$$\begin{array}{r} 17.46 \\ 22.64 \\ 0.75 \\ 31.40 \\ \underline{6.95} \\ 79.20\text{cm} \end{array}$$

(b)
$$\begin{array}{r} 13.02 \\ \underline{-7.35} \\ 5.67 \end{array} \text{ cm}$$

3. 4
4
3
1
3

4. $= 12.8 \times 9.75$
$= 124.800$
$= 125$

(rounded to 3 significant digits)
The area of the floor is 125 m^2.

Page 1

2. (a) four over six

   (b) $\frac{6}{4}$; This expresses the ratio of the number of pies eaten by Henry to those eaten by John.

Exercise 1

1. $\frac{20}{30}$; This is the ratio of the number of loaves baked by the smaller oven to those baked by the larger oven.

Page 2

2. $\frac{3}{2}$; The ratio of the number of rolls of wallpaper hung by Rick to those hung by Chuck.
3. $\frac{10}{17}$; The ratio of the number of letters sent by Alfred to those sent by Jack.
4. $\frac{12}{29}$; The ratio of the length of the seam welded by Tom to that welded by Bill.
5. $\frac{27}{33}$; The ratio of the number of resistors classified by Jerry to those classified by Rudy.
6. $\frac{3}{8}$
7. $\frac{32}{28}$
8. $\frac{30}{29}$

Page 6 **Exercise 2**

Ratios	Cross Products	Conclusion
$\frac{2}{7}, \frac{16}{59}$		
$\frac{3}{4}, \frac{0.15}{0.20}$	$3 \times 0.20 = 0.60$ $4 \times 0.15 = 0.60$	$\frac{3}{4} = \frac{0.15}{0.20}$
$\frac{4}{5}, \frac{16}{50}$	$4 \times 50 = 200$ $5 \times 16 = 80$	$\frac{4}{5} \neq \frac{16}{50}$
$\frac{2}{3}, \frac{10}{15}$	$2 \times 15 = 30$ $3 \times 10 = 30$	$\frac{2}{3} = \frac{10}{15}$
$\frac{5}{9}, \frac{10}{18}$	$5 \times 18 = 90$ $9 \times 10 = 90$	$\frac{5}{9} = \frac{10}{18}$
$\frac{15}{3}, \frac{60}{12}$	$15 \times 12 = 180$ $3 \times 60 = 180$	$\frac{15}{3} = \frac{60}{12}$

Page 8 **Exercise 3**

1.

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2. $32e = 16 \times 14$

$$e = \frac{16 \times 14}{32}$$

$e = 7$

Check

$$\frac{16}{32} = \frac{7}{14}$$

$16 \times 14 = 224$

$7 \times 32 = 224$

3. $36t = 15 \times 84$

$$t = \frac{15 \times 84}{36}$$

$t = 35$

$$\frac{15}{36} = \frac{35}{84}$$

$15 \times 84 = 1260$

$35 \times 36 = 1260$

4. $39t = 26 \times 45$

$$t = \frac{26 \times 45}{39}$$

$t = 30$

$$\frac{26}{30} = \frac{39}{45}$$

$26 \times 45 = 1170$

$39 \times 30 = 1170$

5. $49e = 42 \times 42$

$$e = \frac{42 \times 42}{49}$$

$e = 36$

$$\frac{36}{42} = \frac{42}{49}$$

$36 \times 49 = 1764$

$42 \times 42 = 1764$

6. $15q = 75 \times 12$

$$q = \frac{75 \times 12}{15}$$

$q = 60$

$$\frac{15}{75} = \frac{12}{60}$$

$15 \times 60 = 900$

$12 \times 75 = 900$

7. $80n = 70 \times 56$

$$n = \frac{70 \times 56}{80}$$

$n = 49$

$$\frac{70}{49} = \frac{80}{56}$$

$70 \times 56 = 3920$

$80 \times 49 = 3920$

8. $56i = 35 \times 64$

$$i = \frac{35 \times 64}{56}$$

$$i = 40$$

$$\frac{56}{35} = \frac{64}{40}$$

$$56 \times 40 = 2240$$

$$64 \times 35 = 2240$$

Page 12 **Exercise 4**

%	Ratio	Simplest Form
6%		
20%	$\frac{20}{100}$	$\frac{1}{5}$
67%	$\frac{67}{100}$	$\frac{67}{100}$
125%	$\frac{125}{100}$	$\frac{5}{4}$
98%	$\frac{98}{100}$	$\frac{49}{50}$
17%	$\frac{17}{100}$	$\frac{17}{100}$
43%	$\frac{43}{100}$	$\frac{43}{100}$
32%	$\frac{32}{100}$	$\frac{8}{25}$
70%	$\frac{70}{100}$	$\frac{7}{10}$
35%	$\frac{35}{100}$	$\frac{7}{20}$
28%	$\frac{28}{100}$	$\frac{7}{25}$

Page 16 **Exercise 5**

%	Ratio with denominator of 100	Decimal Number
5%	$\frac{5}{100}$	0.05
33%	$\frac{33}{100}$	0.33
58%	$\frac{58}{100}$	0.58
1%	$\frac{1}{100}$	0.01
137%	$\frac{137}{100}$	1.37
63.5%	$\frac{63.5}{100}$	0.635
83%	$\frac{83}{100}$	0.83
43.2%	$\frac{43.2}{100}$	0.432
250%	$\frac{250}{100}$	2.5
0.2%	$\frac{0.2}{100}$	0.002
7.9%	$\frac{7.9}{100}$	0.079
692%	$\frac{692}{100}$	6.92
44%	$\frac{44}{100}$	0.44
70%	$\frac{70}{100}$	0.7
4.7%	$\frac{4.7}{100}$	0.047

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(b) $\frac{8}{9} = 88.9\%$

Method 1

1. $\frac{8}{9} = \frac{t}{100}$

$9 \times t = 8 \times 100$

$\frac{9 \times t}{9} = \frac{8 \times 100}{9}$

$t = \frac{800}{9}$

$t = 88.9$

2. $\frac{8}{9} = \frac{88.9}{100}$

Method 2

1.

2. $\frac{8}{9} = 0.889$

$0.889 = \frac{88.9}{100} = 88.9\%$

So $\frac{8}{9} = 88.9\%$

Exercise 6

1. $\frac{1}{8} = 12.5\%$

Method 1

$\frac{1}{8} = \frac{n}{100}$

$8n = 100$

$n = \frac{100}{8}$

$n = \frac{25}{2}$

$n = 12.5\%$

Method 2

$$\begin{array}{r} .125 \\ 8\overline{)1.000} \\ \underline{8} \\ 20 \\ \underline{10} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

$\frac{1}{8} = 0.125$

$= \frac{12.5}{100}$

$= 12.5\%$

2. $\frac{3}{4} = 75\%$

Method 1

$\frac{3}{4} = \frac{n}{100}$

$4n = 300$

$n = \frac{300}{4}$

$n = 75\%$

Method 2

$$\begin{array}{r} .75 \\ 4\overline{)3.00} \\ \underline{28} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

$\frac{3}{4} = 0.75$

$0.75 = \frac{75}{100}$

$= 75\%$

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3. $\frac{5}{6} = 83.3\%$

Method 1	Method 2
$\frac{5}{6} = \frac{n}{100}$ $6n = 500$ $n = \frac{500}{6}$ $n = 83.3\%$	$\begin{array}{r} .833 \\ 6\overline{)5.000} \\ \underline{48} \\ 20 \\ \underline{18} \\ 20 \end{array}$ $.833 = \frac{83.3}{100}$ $= 83.3\%$

4. $\frac{2}{3} = 66.7\%$

Method 1	Method 2
$\frac{2}{3} = \frac{n}{100}$ $3n = 200$ $n = \frac{200}{3}$ $n = 66.7\%$	$\begin{array}{r} .666 \\ 3\overline{)2.000} \\ \underline{18} \\ 20 \\ \underline{18} \\ 20 \end{array}$ $.667 = \frac{66.7}{100}$ $= 66.7\%$

5. $\frac{12}{17} = 70.6\%$

Method 1	Method 2
$\frac{12}{17} = \frac{n}{100}$ $17n = 1200$ $n = \frac{1200}{17}$ $n = 70.6\%$	$\begin{array}{r} .1058 \\ 17\overline{)12.000} \\ \underline{119} \\ 100 \\ \underline{85} \\ 150 \end{array}$ $0.706 = \frac{70.6}{100}$ $= 70.6\%$

6. $\frac{5}{7} = 71.4\%$

Method 1	Method 2
$\frac{5}{7} = \frac{n}{100}$ $7n = 500$ $n = \frac{500}{7}$ $n = 71.4\%$	$\begin{array}{r} .7142 \\ 7\overline{)5.000} \\ \underline{49} \\ 10 \\ \underline{7} \\ 30 \\ \underline{28} \\ 20 \end{array}$ $.714 = \frac{71.4}{100}$ $= 71.4\%$

7. $\frac{17}{14} = 121.4\%$

Method 1

$$\frac{17}{14} = \frac{n}{100}$$

$$14n = 1700$$

$$n = \frac{1700}{14}$$

$$n = 121.4\%$$

Method 2

$$\begin{array}{r}
1.214 \\
14\overline{)17.000} \\
\underline{14} \\
3\,0 \\
\underline{2\,8} \\
20 \\
\underline{14} \\
60 \\
\underline{56} \\
4
\end{array}$$

$$1.214 = \frac{121.4}{100}$$

$$= 121.4\%$$

8. $\frac{16}{15} = 106.7\%$

Method 1

$$\frac{16}{15} = \frac{n}{100}$$

$$15n = 1600$$

$$n = \frac{1600}{15}$$

$$n = 106.7\%$$

Method 2

$$\begin{array}{r}
1.0666 \\
15\overline{)16.000} \\
\underline{15} \\
1\,00 \\
\underline{90} \\
100 \\
\underline{90} \\
100
\end{array}$$

$$1.067 = \frac{106.7}{100}$$

$$= 106.7\%$$

Page 2

(b) $\underset{a}{\underline{\quad}}$ is $\underset{r}{\underline{23\%}}$ of $\underset{b}{\underline{82}}$

$$\frac{a}{82} = \frac{23}{100}$$

$$100a = 82 \times 23$$

$$a = \frac{82 \times 23}{100}$$

$$a = 18.86$$

Thus 18.86 is 23% of 82.

Page 3

(b) $\underset{a}{\underline{520}}$ is $\underset{r}{\underline{\quad}}$ % of $\underset{b}{\underline{416}}$

$$\frac{520}{416} = \frac{r}{100}$$

$$416r = 520 \times 100$$

$$r = \frac{520 \times 100}{416}$$

$$r = 125\%$$

520 is 125% of 416.

Page 4

(b) $\underset{a}{\underline{15}}$ is $\underset{r}{\underline{6\%}}$ of $\underset{b}{\underline{\quad}}$

$$\frac{15}{b} = \frac{6}{100}$$

$$6b = 1500$$

$$b = \frac{1500}{6}$$

$$(b) = 250$$

15 is 6% of 250.

Page 7

1. ______________ is $3\frac{1}{4}\%$ of \$548.60.

$$\frac{a}{548.60} = \frac{3.25}{100}$$

$$100a = 548.60 \times 3.25$$

$$a = \frac{548.60 \times 3.25}{100}$$

$$a = 17.8295$$

\$17.83 is $3\frac{1}{4}\%$ of \$548.60.

2. \$45 414 is 120% of ______________.

$$\frac{45414}{b} = \frac{120}{100}$$

$$120\,b = 45414 \times 100$$

$$b = \frac{45414 \times 100}{120}$$

$$b = \$37\ 845$$

\$45 414 is 120% of \$37 845.

3. 13 is ____________ % of 52.

$$\frac{13}{52} = \frac{r}{100}$$

$$52r = 13 \times 100$$

$$r = \frac{13 \times 100}{52}$$

$$r = 25\%$$

13 is 25% of 52.

4. 92 is ____________ % of 12.

$$\frac{92}{12} = \frac{r}{100}$$

$$12r = 92 \times 100$$

$$r = \frac{92 \times 100}{12}$$

$$r = 766\frac{2}{3}\%$$

92 is $766\frac{2}{3}\%$ of 12.

5. ____________ is 375% of 96.

$$\frac{a}{96} = \frac{375}{100}$$

$$100a = 375 \times 96$$

$$a = \frac{375 \times 96}{100}$$

$$a = 360$$

360 metres is 375% of 96 metres.

Page 11

1. (a) Total weight − amount of tin = Amount of lead

   5 − 3.25 = 1.75

   Thus in a 5 kg bar of hard solder, there are 3.25 kg of tin and **1.75** kg of lead.

(b) 1. ______ kg is 38% of 10 kg.

$$\frac{a}{10} = \frac{38}{100}$$

$$100a = 38 \times 10$$

$$a = \frac{38 \times 10}{100}$$

$$a = 3.8$$

There are 3.8 kg of tin in a 10 kg roll of soft solder.

2. Total weight − amount of tin = Amount of lead.

$10 - 3.8 = 6.2$

In a 10 kg roll of soft solder, there are 3.8 kg of tin and 6.2 kg of lead.

2. (a) ______ is 30% of $550.

$$\frac{a}{550} = \frac{30}{100}$$

$$100a = 550 \times 30$$

$$a = \frac{550 \times 30}{100}$$

$$a = 165$$

$165 will be deducted from his check each week.

(b) 550 − 165 = $385.

Eugen takes home $385.

Page 12

3. (a) Total weight = 30 + 1 + 2 + 13 = 46 kg
The casting weighs 46 kg in all.

(b) 13 is ______ % of 46.

$$\frac{13}{46} = \frac{r}{100}$$

$$46n = 13 \times 100$$

$$n = \frac{13 \times 100}{46}$$

$$n = 28$$

28% of the casting is zinc.

(c) 30 is ______ % of 46.

$$\frac{30}{46} = \frac{r}{100}$$

$$46r = 30 \times 100$$

$$r = \frac{30 \times 100}{46}$$

65% of the casting is copper.

4. 420 is 85% of ______.

$$\frac{420}{b} = \frac{85}{100}$$

$$85b = 420 \times 100$$

$$b = \frac{420 \times 100}{85}$$

$$b = 494$$

494 board feet by lumber was delivered to the building site.

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5. (a) 45.5 is 65% of ______.

$$\frac{45.5}{b} = \frac{65}{100}$$

$$65b = 45.5 \times 100$$

$$b = \frac{45.5 \times 100}{65}$$

$$b = 70$$

70 horsepower is the power input.

(b) Power loss = 70 − 45.5
= 24.5

24.5 horsepower is lost in the transmission.

6. Number correct: 80 − 13 = 67

67 is _____% of 80

$$\frac{67}{80} = \frac{r}{100}$$

$$80r = 67 \times 100$$

$$r = \frac{67 \times 100}{80}$$

$$r = 84\%$$

His exam mark was 84%.

7. ______ is $12\frac{1}{2}\%$ of 1600.

$$\frac{a}{1600} = \frac{12.5}{100}$$

$$100a = 12\frac{1}{2} \times 1600$$

$$a = \frac{12.5 \times 1600}{100}$$

$$a = 200$$

200 lbs. of fuel would be used.

Page 14

8. ______ m is 27.3% of 320 m.

$$\frac{a}{320} = \frac{27.3}{100}$$

$$100a = 320 \times 27.3$$

$$a = \frac{320 \times 27.3}{100}$$

$$a = 87$$

He used 87 metres of pipe.

9. Total pay = regular time pay + overtime pay

= \$8.70 × 44 + \$8.70 × 1.5 × 12
= \$382.80 + \$156.60
= \$539.40

$156.60 is ______ % of $539.40.

$$\frac{156.60}{539.40} = \frac{r}{100}$$

$$539.40r = 156.60 \times 100$$

$$r = \frac{156.60 \times 100}{539.40}$$

$$r = 29\%$$

29% of his pay is overtime pay.

10. ______ is 2% of 2000.

$$\frac{a}{2000} = \frac{2}{100}$$

$$100a = 2 \times 2000$$

$$a = \frac{2 \times 2000}{100}$$

$$a = 40$$

40 employees were absent that day.

Page 16 **Example 2:**

(a) $\underset{a}{\_\_}$ is 6.2% of $\underset{b}{\underline{\$8.10}}$

$$\frac{a}{8.10} = \frac{6.2}{100}$$

$$100a = 8.10 \times 6.2$$

$$a = \frac{8.10 \times 6.2}{100}$$

$$a = \$0.50$$

Thus, the welder recieved a $0.50 per hour increase in pay.

Example 3:

(b) $= \$8.10 + \$0.50$
$= \$8.60$

His new wage was $8.60.

Page 17

$$r = \frac{9.50 \times 100}{7.50}$$

$r = 127$

Thus the increase in the number of hours of sunlight was 127%.

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Example 4:

$\underset{a}{\underline{\$156}}$ is $\underset{r}{\underline{\quad}}$ % of $\underset{b}{\underline{\$1040}}$

$$\frac{156}{1040} = \frac{r}{100}$$

$1040r = 156 \times 100$

$$r = \frac{156 \times 100}{1040}$$

$r = 15$

Tony received a 15% discount.

1. (a) ______ is 25% of $5375.

$$\frac{a}{5375} = \frac{25}{100}$$

$100a = 5375 \times 25$

$$a = \frac{5375 \times 25}{100}$$

$a = \$1343.75$

The car depreciates $1343.75 during the first year.

(b) New value = Original cost − depreciation.

= \$5375 − 1343.75
= \$4031.25

Its depreciated value is \$4031.25.

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2. (a) ______ is 450% of \$12 500.

$$\frac{a}{12\ 500} = \frac{450}{100}$$

$$100a = 12\ 500 \times 450$$

$$a = \frac{12\ 500 \times 450}{100}$$

$$a = \$56\ 250$$

The price of the property has increased by \$56 250.

(b) Present value = original price + increase.
= \$12 500 + \$56 250
= \$68 750

The property sells for \$68 750 now.

3. (a) 0.5 is 3% of ____________________.

$$\frac{0.5}{b} = \frac{3}{100}$$

$$3b = 0.5 \times 100$$

$$b = \frac{0.5 \times 100}{3}$$

$$b = 16.7\ \text{dm}$$

The standard measurement of the plank is 16.7 dm.

(b) Length = standard − error

$= 16.7 - 0.5$

$= 16.2$ dm

The plank is actually 16.2 dm long.

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4. 5 volts is $3\frac{1}{2}\%$ of ____________________________.

$$\frac{5}{b} = \frac{3.5}{\begin{array}{c}10\\0\end{array}}$$

$$3.5b = 5 \times 100$$

$$b = \frac{5 \times 100}{3.5}$$

$$b = 143$$

The line voltage is 143 volts.

5. (a) Cost = Price × number

$= \$49.98 \times 15$

$= \$749.70$

The tents cost \$749.70 before the discount.

(b) ____________________________ is 18% of \$749.70

$$\frac{a}{749.70} = \frac{18}{100}$$

$$a = \frac{749.70 \times 18}{100}$$

$$a = \$134.95$$

The discount would be \$134.95.

(c) Total amount to be paid = Cost − discount

= \$749.70 − 134.95

= \$614.75

The Boy Scout troop will have to pay \$614.75 for the tents.

Page 22

Example 1:

Total amount = Principle + Interest

= 8000 + 1960

= 9960

Mr. Smithers paid the bank \$9960 in repayment of his \$8000 loan.

Example 3:

$i =$ ______, p (unpaid balance) = \$432 − **110** = **322**

$r = 1\frac{1}{2}\% = 1.5\% = 0.015$

$i = p \times r \times t$

$i = 322 \times 0.015 \times 1$

$i = 4.83$

On next month's bill, Charlie will have \$4.83 interest due.

Page 23 **Exercise 4**

1. $i = p \times r \times t$ (t = 3 months = $\frac{1}{4}$ year = 0.25 years)
   $i = 1600 \times 0.11 \times 0.25$
   $i = \$44.00$

   Jack will owe \$44.00 in interest.

2. $i = p \times r \times t$
   $i = 13\,000 \times 0.095 \times 3$
   $i = \$3705$

   Total = Principle + Interest

   $= 13\,000 + 3705$
   $= 16\,705$

   At the end of 3 years, Robert will have $16 705.00.

3. Unpaid balance = $835 − 35 = $800

   $i = p \times r \times t$
   $i = 800 \times 0.015 \times 1$
   $i = \$12.00$

   George will have to pay $12.00 in interest next month.

Page 24

4. $i = p \times r \times t$ (t = 18 months = 1.5 years)
   $i = 2000 \times 0.0875 \times 1.5$
   $i = \$262.50$

   Total = Principle + Interest

   $= 2000 + 262.50$
   $= 2262.50$

   After 18 months, Ralph will owe Paul $2262.50.

Page 3

Example 2:

$$\frac{8\frac{1}{2}}{y} = \frac{8000}{2100}$$

$$8000y = 8.5 \times 2100$$

$$y = \frac{8.5 \times 2100}{8000}$$

$$y = \mathbf{2.23}$$

It takes 2.23 hours for the plane to make the 2100 km trip.

Page 4 **Exercise 1**

1. 100 washers → $\frac{5}{8}$ kg

   n washers → 40 kg

$$\frac{100}{n} = \frac{\frac{5}{8}}{40}$$

$$\frac{5}{8}n = 100 \times 40$$

$$n = \frac{100 \times 40}{\frac{5}{8}}$$

$$n = \mathbf{6400}$$

6400 washers will weigh 40 kg.

Page 5

2. $0.008 \rightarrow 6$ cm

   $0.048 \rightarrow n$ cm

   $$\frac{0.008}{0.048} = \frac{6}{n}$$

   $$0.008\,n = 6 \times 0.048$$

   $$n = \frac{6 \times 0.048}{0.008}$$

   $n = \mathbf{36}$

   A 36 cm shaft will have a taper of 0.048 cm.

3. $51 \text{ km} \rightarrow 1\frac{1}{2} \text{ L}$

   $n \text{ km} \rightarrow 15 \text{ L}$

   $$\frac{51}{n} = \frac{1.5}{15}$$

   $$1.5\,n = 51 \times 15$$

   $$n = \frac{51 \times 15}{1.5}$$

   $n = \mathbf{510}$

   The car will travel 510 km on 15 L of gas.

4. 6 feet → $1\frac{1}{2}$ feet

n feet → 45 feet

$$\frac{6}{n} = \frac{1.5}{45}$$

$$1.5n = 6 \times 45$$

$$n = \frac{6 \times 45}{1.5}$$

$n =$ **180 feet**

The building is 180 feet high.

Page 8 **Exercise 2**

There are two different sets of solutions.

1. Length: 7.8 cm or 8.3 cm
2. Length: 3.2 cm or 3.3 cm
3. Length: 6.9 cm or 7.3 cm
4. Length: 2 in or $2\frac{1}{16}$ in
5. Length: 8.7 cm or 9.2 cm

Page 10 **Exercise 3**

1. 3.8 cm
2. $2\frac{9}{10}$ inches
3. $4\frac{3}{16}$ inches
4. 9.2 cm

Page 12 **Exercise 4**

1. (a) 1 cm → 28 km

3.3 cm → n km

$$\frac{1}{3.3} = \frac{28}{n}$$

$n = 28 \times 3.3$

$n =$ **92.4 km** (may vary slightly)

(b) 1 cm → 28 km

2.2 cm → n km

$$\frac{1}{2.2} = \frac{28}{n}$$

$n = 28 \times 2.2$

$n =$ **61.6 km** (may vary slightly)

Page 13

(c) 1 cm → 28 km

1.4 cm → n km

$$\frac{1}{1.4} = \frac{28}{n}$$

$n =$ **39.2 km** (may vary slightly)

(d) 1 cm → 28 km

2.6 cm → n km

$$\frac{1}{2.6} = \frac{28}{n}$$

n = 28 × 2.6

n = **72.8 km** (may vary slightly)

2. Answers to fill in blanks:

③ 24′

Bedroom #1 9′ × 11′

④ 18′

Bedroom #2 9′ × 12′

Bedroom #3 13′ × 14′ ⑤ 15′

⑥ 24′

Page 14

2. 1 inch → 10 feet

4.2 inch → n feet

$$\frac{1}{4.2} = \frac{10}{n}$$

n = 10 × 4.2

n = **42 feet**

3. 1 inch → 10 feet

2.4 inches → n feet

$$\frac{1}{2.4} = \frac{10}{n}$$

n = 10 × 2.4

n = **24 feet**

4. 1 inch → 10 feet

1.8 inches → n feet

$$\frac{1}{1.8} = \frac{10}{n}$$

$n = 10 \times 1.8$

$n =$ **18 feet**

5. 1 inch → 10 feet

1.5 inches → n feet

$$\frac{1}{1.5} = \frac{10}{n}$$

$n = 10 \times 1.5$

$n =$ **15 feet**

6. 1 inch → 10 feet

2.4 inches → n feet

$$\frac{1}{2.4} = \frac{10}{n}$$

$n = 10 \times 2.4$

$n =$ **24 feet**

Page 15

Bedroom No. 2

1 inch → 10 feet

0.9 inches → n feet

$$\frac{1}{0.9} = \frac{10}{n}$$

$n = 10 \times 0.9$

$n =$ **9 feet**

1 inch → 10 feet

1.2 inches → n feet

$$\frac{1}{1.2} = \frac{10}{n}$$

$n = 10 \times 1.2$

$n =$ **12 feet**

Living Room

1 inch → 10 feet

1.3 inches → n feet

$$\frac{1}{1.3} = \frac{10}{n}$$

$n = 10 \times 1.3$

$n =$ **13 feet**

1 inch → 10 feet

1.4 inches → n feet

$$\frac{1}{1.4} = \frac{10}{n}$$

$n = 10 \times 1.4$

$n =$ **14 feet**

Page 16

Dining Room

1 inch → 10 feet

0.9 inch → n feet

$$\frac{1}{0.9} = \frac{10}{n}$$

$n = 10 \times 0.9$

$n =$ **9 feet**

1 inch → 10 feet

1.1 inches → n feet

$$\frac{1}{1.1} = \frac{10}{n}$$

$n = 10 \times 1.1$

$n =$ **11 feet**

In each of the above calculations, the scale measurement was multiplied by what number? **10**

Page 17

1. 10 mm → 8 cm

   11 mm → n cm

   $\frac{10}{11} = \frac{8}{n}$

   $10n = 8 \times 11$

   $n = \frac{8 \times 11}{10}$

   $n = \mathbf{8.8\ cm}$

2. 10 mm → 8 cm

   26 mm → n cm

   $\frac{10}{26} = \frac{8}{n}$

   $10n = 8 \times 26$

   $n = \frac{8 \times 26}{10}$

   $n = \mathbf{20.8\ cm}$

3. 10 mm → 8 cm

   90 mm → n cm

   $\frac{10}{90} = \frac{8}{n}$

   $10n = 8 \times 90$

   $n = \frac{8 \times 90}{10}$

   $n = \mathbf{72\ cm}$

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(c) $\frac{\frac{1}{10}}{f} = \frac{1}{2}$

$\mathbf{f = \frac{1}{10} \times 2}$

$\mathbf{f = \frac{2}{10}}$

Thus, the width of the flower bed should be drawn $\frac{2}{10}$ inches wide on the diagram.

(e) $g = \frac{1}{10} \times 25$

$= \frac{25}{10}$ **inches**

Thus, the width of the garden should be drawn as $2\frac{1}{2}$ inches on the diagram.

Exercise 5 – Calculations

10 yards → 1 cm

110 yards → n cm

$\frac{10}{110} = \frac{1}{n}$

$10\,n = 110$

$n = \frac{110}{10}$

n = **11 cm**

10 yards → 1 cm

65 yards → n cm

$\frac{10}{65} = \frac{1}{n}$

$10\,n = 65$

$n = \frac{65}{10}$

n = **6.5 cm**

10 yards → 1 cm

25 yards → n cm

$\frac{10}{25} = \frac{1}{n}$

$10n = 25$

$n = \frac{25}{10}$

n = **2.5 cm**

10 yards → 1cm

5 yards → n cm

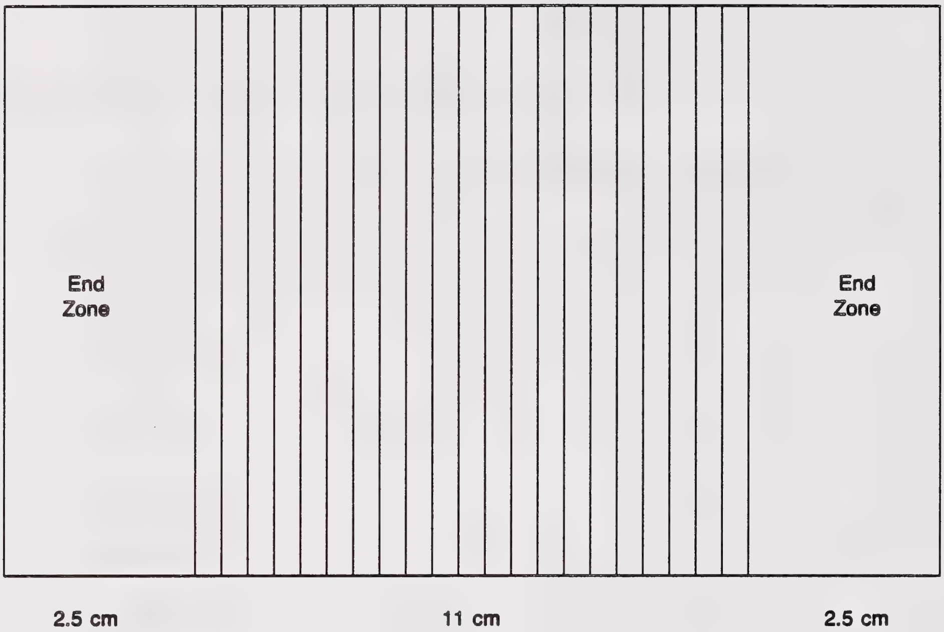
$\frac{10}{5} = \frac{1}{n}$

$10n = 5$

$n = \frac{5}{10}$

n = **0.5 cm**

Diagram



Page 2

Examples:

1. $\frac{3}{4} = \frac{x}{5}$

$4x = 3 \times 5$

$4x = 15$

$x = \frac{15}{4}$

$x = 3\frac{3}{4}$

It will take the 4 men $3\frac{3}{4}$ hours to build the brick wall.

Page 4 **Exercise 1**

1. $3\frac{1}{2}$ hours x km/hr

$5\frac{1}{2}$ hours 80 km/hr

$$\frac{3\frac{1}{2}}{5\frac{1}{2}} = \frac{x}{80}$$

$$5\frac{1}{2}x = 3\frac{1}{2} \times 80$$

$$x = \frac{3\frac{1}{2} \times 80}{5\frac{1}{2}}$$

$x = \mathbf{50.9}$

His average speed on the return trip was 50.9 km/hr.

Page 5

2. 6 men n hr

10 men 12 hr

$$\frac{6}{10} = \frac{n}{12}$$

$$10n = 12 \times 6$$

$$n = \frac{12 \times 6}{10}$$

$$n = 7.2$$

It will take 7.2 hours for the 10 men to complete the job.

3. 3 ohm n amps

50 ohms 15 amps

$$\frac{3}{50} = \frac{n}{15}$$

$$50n = 3 \times 15$$

$$n = \frac{3 \times 15}{50}$$

$$n = \frac{9}{10} \text{ amp}$$

The 50 ohm resistor will pass $\frac{9}{10}$ amp.

Page 6

Example:

$$\frac{72}{y} = \frac{1000}{750}$$

$$1000y = 72 \times 750$$

$$y = \frac{72 \times 750}{1000}$$

$$y = 54$$

The second gear has 54 teeth.

Example:

$$\frac{3}{8} = \frac{3}{1500}$$

$$8z = 3 \times 1500$$

$$z = \frac{3 \times 1500}{8}$$

$$z = 562.5$$

So the grinder pulley rotates at 562.5 r.p.m.

Page 8

Example:

$$\frac{4.5}{y} = \frac{1.5}{6}$$

$$1.5y = 4.5 \times 6$$

$$y = \frac{4.5 \times 6}{1.5}$$

$$y = 18$$

18 pounds of force were needed.

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$$\frac{z}{25} = \frac{1}{6}$$

$$6z = 25$$

$$z = \frac{25}{6}$$

$$z = 4\frac{1}{6}$$

With the 25 pounds of force, Frank can lift a $4\frac{1}{6}$ pound fish from the water.

Exercise 2

1. 3 in n lb.

   12 in 300 lb.

   $$\frac{3}{12} = \frac{n}{300}$$

   $$12n = 3 \times 300$$

   $$n = \frac{3 \times 300}{12}$$

   $$n = 75 \text{ lb}$$

   75 lbs. of force is needed to remove the nail.

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2. 18 teeth x r.p.m.

   36 teeth 2300 r.p.m.

   $$\frac{18}{36} = \frac{x}{2300}$$

   $$36x = 18 \times 2300$$

   $$x = \frac{18 \times 2300}{36}$$

   $$x = 1250$$

   The camshaft will turn at 1250 r.p.m.

Note: Distance is always measured from the fulcrum. The distance from his right hand (fulcrum), to the resistance is $2' + 2\frac{1}{2}' = 4\frac{1}{2}'$

3. 2' 30 lbs.

$4\frac{1}{2}'$ n lbs.

$$\frac{2}{4\frac{1}{2}} = \frac{30}{n}$$

$$2n = 4\frac{1}{2} \times 30$$

$$n = \frac{4\frac{1}{2} \times 30}{2}$$

$$n = 67.5 \text{ lbs.}$$

Jack must apply 67.5 pounds of force to lift a 30 pound shovel of snow.

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4. 4.8 cu. in. n psi.

0.6 cu. in. 14 psi.

$$\frac{4.8}{0.6} = \frac{n}{14}$$

$$0.6n = 4.8 \times 14$$

$$n = \frac{4.8 \times 14}{0.6}$$

$$n = 112 \text{ psi}$$

The pressure at the top of the stroke is 112 pounds per square inch.

5. 2’ x lb.

6’ 250 lbs.

$$\frac{2}{6} = \frac{x}{250}$$

$$6x = 2 \times 250$$

$$x = \frac{2 \times 250}{6}$$

$$x = 83\frac{1}{3} \text{ lb.}$$

Don must apply $83\frac{1}{3}$ lb. of force to pick up the wheelbarrow.

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A gear with 48 teeth is meshed with a gear having 30 teeth.

Page 13

$$\frac{48}{30} = \frac{1600}{y}$$

$$48y = 30 \times 1600$$

$$y = \frac{30 \times 1600}{48}$$

$$y = 1000$$

Thus the 8” gear makes 1000 r.p.m.

Exercise 3

1. increase
   direct

   3 hr. → 756 sq. ft.
   n hr. → 2100 sq. ft.

   $$\frac{3}{n} = \frac{756}{2100}$$

   $$756n = 3 \times 2100$$

   $$n = \frac{3 \times 2100}{756}$$

   $$n = 8.3 \text{ hr.}$$

   It will take the two men 8.3 hrs. to paint 2100 sq. ft. of wall area.

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2. decrease
   increase

   16 hrs. 3 men
   x hrs. 1 man

   $$\frac{16}{x} = \frac{3}{1}$$

   $$3x = 16 \times 1$$

   $$x = \frac{16 \times 1}{3}$$

   $$x = 5\frac{1}{3}$$

   It would take the 3 men $5\frac{1}{3}$ hours to build the walk.

3. more
direct

6.3 km   1 litre
248   n litres

$$\frac{6.3}{248} = \frac{1}{n}$$

$$6.3n = 248$$

$$n = \frac{248}{n}$$

$$n = 39.4 \text{ litres}$$

Steve will need 39.4 litres for the trip to Slave Lake.

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4. slower
inverse

1800 r.p.m.   7 in.
n r.p.m.   3 in.

$$\frac{1800}{n} = \frac{7}{3}$$

$$7n = 3 \times 1800$$

$$n = \frac{3 \times 1800}{7}$$

$$n = 771$$

The driven pulley will rotate at 771 r.p.m. or 800 r.p.m. (rounded to nearer 100 r.p.m.)

5. decrease
   direct

   1200 r.p.m. 1000 r.p.m.
   3200 r.p.m. n r.p.m.

   $$\frac{1200}{3200} = \frac{1000}{n}$$

   $$1200n = 3200 \times 1000$$

   $$n = \frac{3200 \times 1000}{1200} = 2666.67$$

   $$n \doteq 2700 \text{ r.p.m.}$$

   The driven pulley now rotates at 2700 r.p.m. (rounded to nearer 100.)

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6. decrease
   increase

   5 hrs. 90 km/hr.
   n hrs. 64 km/hr.

   $$\frac{5}{n} = \frac{90}{64}$$

   $$90n = 5 \times 64$$

   $$n = \frac{5 \times 64}{90}$$

   $$n = 3\frac{5}{9} \text{ hrs.}$$

   Hank spent $3\frac{5}{9}$ hours on the road.

7. (a) increase
direct

808 km	10 hr.
432 km	n hr.

$$\frac{808}{432} = \frac{10}{n}$$

$$808n = 432 \times 10$$

$$n = \frac{432 \times 10}{808}$$

$$n = 5.3 \text{ hr.}$$

It will take 5.3 hrs. to go from Kamloops to Vancouver.

(b) Total time = 10 hr. + 5.3 hr.
= 15.3 hrs.

Totally, Joe and Sarah will spend 15.3 hours on the road.

Powers and Square Roots

Page 3 **Exercise 1**

1. (a) $= (3 \times 3) = 9$
(b) $= (1 \times 1 \times 1) = 1$
(c) $= (2 \times 2 \times 2 \times 2) = 16$
(d) $= 4 \times 4 = 16$
(e) $= 5 \times 5 \times 5 = 125$
(f) $= 3 \times 3 \times 3 \times 3 = 81$

2. (a) $= a^3$
(b) $= r^2$
(c) $= s^3$
(d) $= 19^5$
(e) $= 21^3$
(f) $= 4^6$

Page 4

Number	Square of Number	Factors	Product (Perfect Square)
4	4^2	4 × 4	16
5	5^2	5 × 5	25
6	6^2	6 × 6	36
7	7^2	7 × 7	49
8	8^2	8 × 8	64
9	9^2	9 × 9	81
10	10^2	10 × 10	100
11	11^2	11× 11	121
12	12^2	12 × 12	144
13	13^2	13 × 13	169
14	14^2	14 × 14	196
15	15^2	15 × 15	225
16	16^2	16 × 16	256
17	17^2	17 × 17	289
18	18^2	18 × 18	324
19	19^2	19 × 19	361
20	20^2	20 × 20	400
21	21^2	21 × 21	441
22	22^2	22 × 22	484
23	23^2	23 × 23	529
24	24^2	24 × 24	576
25	25^2	25 × 25	625

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1.	= 4	= 400	= 40 000
2.	= 25	= 2500	= 250 000
3.	= 196	= 19 600	= 1 960 000

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Examples:

$= 5.243 \times 10^{12}$

Since the decimal point moves **12** places to the left, the exponent is **12**.

Examples:

Nine zero placeholders are needed to enable the decimal to move **12**places to the right.

Page 9 **Exercise 2**

1. $= 125$
2. $= 19 \times 19 = 361$
3. $= 31 \times 31 = 961$
4. $= 25 \times 25 = 625$
5. $= 50 \times 50 = 2500$
6. $= 6 \times 6 \times 6 = 216$
7. $= 1.2 \times 10^3$
8. $= 1.73 \times 10^3$
9. $= 5.263 \times 10^9$
10. 9.834×10^3
11. 6.59×10^7
12. $= 7.26 \times 10^2$
13. = 183 000 000 000
14. = 5 400 000
15. = 129.34
16. = 269
17. = 74 260
18. = 4820

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Perfect Square	**Perfect Square as a Power**	**Square Root**
0	0^2	$\sqrt{0^2} = 0$
1	1^2	$\sqrt{1^2} = 1$
4	2^2	$\sqrt{2^2} = 2$
9	3^2	$\sqrt{3^2} = 3$
16	4^2	$\sqrt{4^2} = 4$
25	5^2	$\sqrt{5^2} = 5$
36	6^2	$\sqrt{6^2} = 6$
49	7^2	$\sqrt{7^2} = 7$
64	8^2	$\sqrt{8^2} = 8$
81	9^2	$\sqrt{9^2} = 9$
100	10^2	$\sqrt{10^2} = 10$
121	11^2	$\sqrt{11^2} = 11$
144	12^2	$\sqrt{12^2} = 12$
169	13^2	$\sqrt{13^2} = 13$
196	14^2	$\sqrt{14^2} = 14$
225	15^2	$\sqrt{15^2} = 15$
256	16^2	$\sqrt{16^2} = 16$
289	17^2	$\sqrt{17^2} = 17$
324	18^2	$\sqrt{18^2} = 18$
361	19^2	$\sqrt{19^2} = 19$
400	20^2	$\sqrt{20^2} = 20$
441	21^2	$\sqrt{21^2} = 21$
484	22^2	$\sqrt{22^2} = 22$
529	23^2	$\sqrt{23^2} = 23$
576	24^2	$\sqrt{24^2} = 24$
625	25^2	$\sqrt{25^2} = 25$

Exercise 3

1. $\sqrt{256} = 16$ $\sqrt{25\,600} = 160$ $\sqrt{256\,000\,000} = 16\,000$
2. $\sqrt{169} = 13$ $= 13\,000$ $= 130\,000$
3. $= 23$ $= 230$ $= 23\,000$

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4. $= 19$ $= 190$ $= 1900$
5. $= 2$ $= 20$ $= 20\,000$

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Thus $\sqrt{37}$ ◦ **6.083**

Page 15 **Exercise 4**

1. **Note:** If you have a calculator available then you may use it to find square roots.

2. $\doteq 4.359$

$$\begin{array}{r} 4.359 \\ \times 4.359 \\ \hline 39231 \\ 21795 \\ 13077 \\ 17436 \\ \hline 19.000881 \end{array}$$

3. $\doteq 9.338$

$$\begin{array}{r} 9.338 \\ \times 9.338 \\ \hline 74704 \\ 28014 \\ 28014 \\ 84042 \\ \hline 87.198244 \end{array}$$

4. $\doteq 1.517$

$$\begin{array}{r} 1.517 \\ \times 1.517 \\ \hline 10619 \\ 1517 \\ 7585 \\ 1517 \\ \hline 2.301289 \end{array}$$

5. $\doteq 5.348$

$$\begin{array}{r} 5.348 \\ \times 5.348 \\ \hline 42784 \\ 21392 \\ 16044 \\ 26740 \\ \hline 28.601104 \end{array}$$

6. $\doteq 3.142$

$$\begin{array}{r}
3.142 \\
\times 3.142 \\ \hline
6284 \\
12568 \\
3142 \\
9426 \\ \hline
9.872164
\end{array}$$

7. $\doteq 8.544$

$$\begin{array}{r}
8.544 \\
\times 8.544 \\ \hline
34176 \\
34176 \\
42720 \\
68352 \\ \hline
72.999936
\end{array}$$

8. $\doteq 7.681$

$$\begin{array}{r}
7.681 \\
\times 7.681 \\ \hline
7681 \\
61448 \\
46086 \\
53767 \\ \hline
58.997761
\end{array}$$

Page 22 **Exercise 5**

1. $19.02 \doteq 19.0$

$$\begin{array}{r|l}
1 & 362.00 \\
 & \underline{1\times} \\
2\underline{9} & 262 \\
 & \underline{261} \\
38\underline{0} & 100 \\
 & \underline{0} \\
380\underline{2} & 10000 \\
 & \underline{7604} \\
 & 2396
\end{array}$$

2. $11.54 \doteq 11.5$

$$\begin{array}{r|l}
 & 133.26 \\
 & \underline{1} \\
2\underline{1} & 033 \\
 & \underline{21} \\
22\underline{5} & 1226 \\
 & \underline{1125} \\
230\underline{4} & 10100 \\
 & \underline{9216} \\
 & 884
\end{array}$$

3. $8.26 \doteq 8.3$

$$\begin{array}{r|l}
8 & 68.263 \\
 & \underline{64} \\
16\underline{2} & 426 \\
 & \underline{324} \\
164\underline{6} & 10230 \\
 & \underline{9876} \\
\underline{} & 354
\end{array}$$

4. $35.01 \doteq 35.0$

$$\begin{array}{r|l}
3 & 1226 \\
 & \underline{9} \\
6\underline{5} & 326 \\
 & \underline{325} \\
70\underline{0} & 100 \\
 & \underline{0} \\
700\underline{1} & 10000 \\
 & \underline{7001} \\
 & 2999
\end{array}$$

NOTE: You may use a calculator to find square roots.

N.L.C. - B.N.C.
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